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The role of production subsidies in general equilibrium macroeconomic models with imperfect competition

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Declaration

I declare that I am the author of this thesis and that I have consulted all the references cited. All the work of which this thesis is a record has been done by myself and has not been previously accepted for a higher degree.

Chang Yee Kwan
Ph.D Candidate
Date ....

Certification

I certify that Mr. Chang Yee Kwan conducted his research under my supervision in the Department of Economic Studies, University of Dundee. Mr. Kwan has fulfilled all the conditions of the relevant Ordinances and Regulations of the University of Dundee for obtaining the Degree of Doctor of Philosophy.

Catia Montagna
Professor of Economics
Date ....

Abstract

Industrial policy in the form of direct and indirect government subsidy provision to firms in specific sectors of the economy is a common sight in many countries. Some of the most often quoted examples are East Asian economies such as Japan and Taiwan. While industrial policy is touted as a possible engine to generate economic growth, empirical validations on the benefits from subsidy provisions have been mixed. It is often argued that a policy of non-intervention by the government may appear to be the optimal policy to pursue.

However, this contrasts with historical observations of regular government subsidy provisions to firms in many countries. This thesis constructs a two-sector non-monetary macroeconomic model with monopolistic competition to examine welfare and other related effects of a subsidy provision by the government to firms financed by a proportional income tax. Firms in the monopolistically competitive sector receive a subsidy in the form of lump sum transfers or as some proportion of a variable cost component while firms in the perfectly competitive sector do not.

The analysis is first carried out in an economy where labour supply is assumed to be exogenous and perfectly inelastic. This serves to provide a simple and clear exposition on the effects of a subsidy provision and to serve as a benchmark analysis to build upon. This is subsequently extended by allowing for labour supply to be endogenously determined to examine labour market effects of subsidy policies. The implications of subsidy provisions in the presence of international trade are studied by constructing a small open economy model where the effects of any policy implementation do not affect world prices or income.

The principle findings we obtain are that when monopolistically competitive

firms receive a cost-reducing subsidy, welfare improvements are always possible regardless of which cost variable the government subsidises. Furthermore, there is always a positive optimal subsidy which raises social welfare. When the supply of labour is endogenous, the corresponding tax imposed on income will always induce an increase in labour supply. Trade is shown not to affect the principle findings: there remains an optimal level of subsidy which is Pareto-improving. A further implication in the open economy context is that the subsidy acts as a form of import-substitution and export-promotion instrument which potentially alters the domestic economy's trade patterns.

Chapter 1

Introduction

1.1. Preliminaries

The broad theme of this thesis comes under the term 'industrial policy'. As Pack and Saggi (2006) note, this term often implies different issues to different people and it is difficult to give an exact definition to it. We are not proposing any particular definition for industrial policy, since this is not what we are most concerned with here. However, to avoid any definition ambiguities for the rest of this thesis, we follow Pack and Saggi (2006) in defining it as "any type of selective government intervention or policy that attempts to alter the structure of production in favour of sectors……in a way that would not occur in the absence of such intervention in the market equilibrium."

We also clarify several other terms and characteristics which are used throughout this thesis. The economy we construct is a non-monetary economy and produces two classes of goods, an A- and a Y-good. The A-good is a homogeneous commodity which is produced and traded under perfect competition. This reflects goods that are very close substitutes and (potentially) low-skilled in production such that the quantity of factor inputs correspond directly to the quantity of output. Sectors that fit this description include the agricultural sector and low-skilled industries like garment manufacturing. As such, we will use the term 'agriculture' alongside 'homogeneous' to refer to the A-good.

The Y-good is a composite of differentiated varieties that have increasing returns to scale in production. This is synonymous with industrial sectors such as silicon wafers and chemicals which require some considerable level of fixed or start-up costs before production can take place[1]. Thus, the terms ‘industry’, ‘differentiated’ and ‘manufacturing’ are taken as equivalent and will be used interchangeably to mean the Y-good. In our context, industrial policy thus refers to policy aimed at influencing the development of this sector of the economy.

We shall further assume throughout that the economy consists of only one factor of production, labour. Labour is assumed to be homogeneous and mobile between sectors and paid a wage, w. Wages derived from the supply of labour make up the sole source of income in this economy. We also make an implicit assumption that the government intends for the Y-sector[2] to expand and therefore, provides Y-firms with some form of production subsidy. Finally, the model is a static one and the government always practices a balanced budget.

1.1.1. Objective

Carlsson (1983) offers a list of a series of policy instruments which can potentially come under the term of industrial policy which includes, but is not restricted to, prohibitive tariffs, government ownership, labour market policy, etc. Specifically, what we are interested in exploring in this thesis is the use of industrial subsidies. We define subsidies to include different forms of transfer payments, denominated in monetary terms or in terms of the numeraire good, made by the government to firms and that contribute to a reduction in the firms’ costs of production. We want to explore the possible effects that arise as a result of the provision of such transfers from the government to firms. These include effects on the level of labour supply as

[1] While monopolistic competition was credited to both Edward Chamberlin and Joan Robinson (Bellante, 2004), Harrod (1930) had previously provided a more methodological exposition of the effects that can arise from firms experiencing increasing returns to scale in production.
[2] This could perhaps stem from part of an industrialisation drive or some belief that there are benefits to be gained from expanding the Y-sector.

well as the allocative outcomes, the effect of subsidy provision on the level of total output in the *Y*-sector and welfare. We shall also consider whether there any optimal policy rules.

Much of the existing economic literature, as recently surveyed by Pack and Saggi (2006), is concerned with empirical effects of subsidy provisions. To the best of our knowledge, the theoretical literature focussing on how the government can influence the level of production costs via the use of subsidies is somewhat less developed. A considerable amount of effort in the theoretical literature has been directed to study the use of fiscal policy in the economy but appears to be aimed to analysing the resulting outcomes when the government plays the role of a consumer, or where government expenditure is used for the purpose of aggregate demand management. Caraballo and Usabiaga (2006) provide a brief survey of some of the literature related to this aspect of fiscal policy.

Thus, this thesis contributes to fill some of the existing gap in the existing theoretical literature on the effects subsidies can have in a wider macroeconomic setting. Specifically, our findings add on to the current literature in three ways:

i. We show that monopolistic competition provides an avenue for a legitimate use of a tax-subsidy policy combination by a welfare-maximising government.

ii. We illustrate and, where possible, derive the welfare-maximising level of subsidy to be given by the government. This is found always to be positive.

iii. We find that supply side policies which reduce production costs of firms in the increasing returns to scale sector unambiguously yield a Pareto improvement.

1.1.2. Structure

This thesis consists of 5 chapters and is presented such that Chapters 2, 3 and 4 can each be read individually if desired without any significant loss of continuity.

The purpose of this chapter is to provide an introduction and the motivation for the thesis. The model which we subsequently use for our analysis in the rest of this thesis is based on the Dixit and Stiglitz (1977) model of monopolistic competition. In what follows, Section 1.2 puts forward the motivation for this thesis. We discuss the choice of the modelling technique briefly in Section 1.3. Section 1.4 covers briefly some literature on the use of fiscal policy in a monopolistically competitive economy. Section 1.5 concludes the chapter.

The structure and coverage of the rest of the thesis is as follows. In Chapter 2, we construct a one-factor, two-good (sector) general equilibrium macroeconomic model of a closed economy based on Dixit and Stiglitz (1977). Consumers derive utility from the consumption of the two types of goods, a homogeneous good and a differentiated one. The former is produced under perfectly competitive conditions while firms producing the latter have decreasing average costs of production. Labour wages are the only source of income in this economy. This chapter serves two purposes. The first is to examine if industrial policy in the form of subsidy provisions to the differentiated goods sector is welfare improving. The second is to derive and examine an optimal policy rule (if it exists). This serves as a benchmark analysis upon which the subsequent chapters will be built. We consider the use of three types of subsidy policies: a lump-sum transfer to firms, a per-unit production payment, and a subsidy to the average costs of production. In all cases, the subsidy is financed by a proportional tax on income.

Chapter 3 expands on the results from the benchmark case. We extend the model from the previous chapter by allowing the individual to vary his labour supply decision. There are two rationales for doing so. Allowing for an endogenously determined labour supply introduces a greater degree of realism and practical relevance into the model. Secondly, this provides us with the opportunity to examine the effect of the interaction between taxes and subsidies on labour supply – as a measure of the level of *economic activity* (or the degree of participation in the economy). The results derived from this chapter can provide insights that could potentially be useful for the purpose of labour market policy formulation.

The analysis of both Chapters 2 and 3 are conducted within the context of a closed economy. However, the presence of large volumes of observed cross-border flows of goods and services between countries on a regular basis gives the motivation for Chapter 4. We thus extend the benchmark case by incorporating the presence of a foreign sector into the model. The domestic economy is modelled under the assumption that it is a price-taker in the world market, i.e. a small open economy. The introduction of trade flows into the analysis provides an avenue to examine the role that subsidies play in influencing the pattern of trade and welfare.

Chapter 5 concludes the thesis by recapitulating the primary findings together with a brief interpretation of the obtained results for possible policy relevance. Potential avenues for future research are also broached upon and suggested here.

1.2. Motivation

A story one hears regularly is how East Asian economies such as Japan, Singapore and Taiwan have experienced high economic growth as a result of heavy state involvement. For example, Lim (1991) documents some of the different ways by which public sector intervention, or ‘industrial targeting’, was an integral part of the industrialisation process in these countries. Some of the variety of mechanisms the government uses comes in forms such as the provision of government-backed credit financing facilities, tax holidays and various trade instruments such as import tariffs or quotas and export subsidies. These are designed such that they provide incentives for the creation of an import-substituting or export-promoting industrial sector[3].

Yet, such policies are not confined to just East Asia. There is a large literature documenting the existence and use of industrial policies in various forms, as well as another set which analyses their use in a variety of scenarios. Sweden for example, had used subsidy payments to industrial manufacturing companies as a macroeconomic stabilisation tool (Carlsson, 1983). Santarelli and Vivarelli (2002) highlight the case of the Italian electronics sector where firms are provided with subsidies to encourage entry into the industry. Even for an economy that purportedly does not have a culture of an active industrial policy, Kehtels (2007) reports that large amounts of Federal targeted funding (to the tune of over US$40 billion annually) are continually provided in areas such as research and development and enterprise development in the USA.

Apart from subsidies to existing industries, a study by Charlton (2003) notes that the chase for foreign direct investment between governments has given rise to situations where “Common investment incentive instruments include cash grants, corporate tax reductions, property tax abatements, sales tax exemptions, loans, loan guarantees, assistance with firm-specific job training funds and infrastructure subsidies”. This suggests therefore that subsidies are not an unimportant part of industrial policy, empirically speaking[4].

However, one must question the wisdom of subsidy provisions. Pack and Saggi

[3] Examples which come to mind here include the automobile and semiconductor industries.
[4] As a digression, agricultural subsidies also form a substantial part of subsidy payments. An analysis of the welfare effects of agricultural subsidies can be found in Koo and Kennedy (2006). However, our focus here is with subsidies that are provided for the objective of some form of industrialisation policy.

(2006) address this issue by considering a series of market imperfections such as economies of scale in production and information and coordination failures as a theoretical basis and justification for governments to give out subsidies to firms. The presence of any of these characteristics signals a possible legitimacy for government action. Their conclusion, based on a review of the academic empirical literature, suggests that there stands no real justification for a government to intervene. Using measurement indicators such as total factor productivity, firm size and industry growth rates to analyse the impact of subsidies, it appears that the provision of subsidies give no added benefit. No intervention is likely to be the optimal policy stance to take.

Why then, do subsidies continue to be used as a policy tool if they do not yield any benefit? Is there some theoretical backing which justifies this or are there other effects which give welfare gains to society but these are not captured in numerical indicators such as productivity and growth rates? This apparent conflict between casual observation of government behaviour and the results from empirical analyses gives us ample motivation to want to examine the effect of subsidies in greater detail in this thesis.

1.3. Modelling Methodology

1.3.1. Market Structure

The market structure we adopt for our analytical model is based on the Dixit and Stiglitz (1977) model of monopolistic competition. There are several reasons for this choice. Firstly, as mentioned in Neary (2003), it is generally agreed that in reality, product markets are rarely perfectly competitive and that there are some degree of (internal) scale economies present. Secondly, in many of the papers mentioned, the industry generally refers to either a host of firms within a particular industry sector or a combination of various industry sectors in the economy, all of whom involve some form of initial start-up or fixed costs that are not insignificant. Finally, in order to analyse the effects of subsidies fully, the market structure should be sufficiently tractable as to be embedded within a general equilibrium framework.

Monopolistic competition presents itself as a suitable candidate as, following on from Chamberlin (1951), firms in this type of market form are assumed each to produce a good that is slightly different from the next, so each firm has some degree of monopoly power over its own product. Yet, the number of firms in the market is taken as sufficiently large to render any individual firm incapable of influencing market prices and with no consideration of any strategic interaction between firms[5].

While it could be argued that oligopoly would be a more ideal market structure, it has proven difficult to model it within a general equilibrium framework. One main obstacle to this is the possible absence of a stable equilibrium outcome as shown by Roberts and Sonnenschein (1977)[6]. Conversely, the assumptions that characterise monopolistic competition allow for the derivation of a stable equilibrium, making it ideal for our purpose here.

1.3.2. Model

Within the existing economic literature, there are a number of ways in which monopolistic competition is modelled. Norman and Thisse (1994) have assembled a collection comprising of the various contributions to this field which the interested reader is referred to. Within the existing economic literature, the Dixit and Stiglitz (1977) model has become the 'workhorse' model of monopolistic competition

[5] This can be argued as being valid in reality since industrial targeting implies that firms within the same targeted industry benefit collectively from the policy, or targeting is directed at distinct industry sectors altogether, such that there is little actual strategic interaction between them overall.
[6] Neary (2003) provides for a more complete discussion about the problems related to this.

largely because of its tractability in deriving explicit solutions.

It should be mentioned here that the Spence (1976b) and Dixit and Stiglitz (1977) models are relatively similar. Brakman and Heijdra (2004) provide a very accessible review to which we refer the reader to. However, we highlight here why the Dixit and Stiglitz (1977) model is more appropriate for use between the two.

In both models, firms producing the differentiated good incur some level of fixed costs and a constant unit-specific marginal cost in production. This results in firms having increasing returns to scale, or decreasing average costs, of production. Each firm's good is an imperfect substitute to the next with a constant elasticity of substitution between each variety. The differentiated good is modelled as a CES function in the form

$$Y = \left(\int_0^N y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1.1}$$

where the elasticity of substitution, σ, is $\sigma > 1$[7].

However, Spence (1976b) models the individual's utility function to be quasi-linear[8]. Dixit and Stiglitz (1977) take instead, the utility function to be of the form

$$U = Y^{\mu} A^{1-\mu}, \tag{1.2}$$

where Y refers to the composite of differentiated goods, A is the homogeneous good, and μ and $(1-\mu)$ are the consumption shares of income to each class of good. This minor difference in formulation yields comparatively similar outcomes with respect to the Y-good, but the removal of any income effects renders the Spence (1976b)

[7] Note that Dixit and Stiglitz (1977) utilised a discrete *specification*. However, given that N is assumed to be large in the Chamberlinian model, the continuous specification has become common in the macroeconomics, growth and trade literature: e.g. Dixit and Stiglitz (1974), reprinted in Braakman and Heijdra (2004); See also the discussion of the Dixit-Stiglitz model in Fujita *et al* (1999) for details. Clearly in the continuous specification, N does not define a number of firms, but a mass of firms.

[8] Spence (1976b) does not model this formally, but states that he "shall explicitly assume away income effects" from the onset.

analysis to be more partial equilibrium in nature[9]. Allowing for both income and substitution effects in both goods makes the Dixit and Stiglitz (1977) model more suited to a general equilibrium analysis.

An example of an alternative modelling approach is the seminal contribution of Lancaster (1980) which follows on from his earlier work[10]. However, it is comparatively more unwieldy when placed within a general equilibrium framework as compared to the framework developed by Spence (1976b) and Dixit and Stiglitz (1977) and we shall not cover it here[11]. The interested reader is referred to Lancaster (1980) and the collection by Norman and Thisse (1994) for coverage of the relevant literature. Thence, in line with much of the existing economic literature, we will use the Dixit-Stiglitz model for our analytical purposes in this thesis.

1.4. Preliminary Background Literature

A relatively substantial quantity of literature has arisen incorporating monopolistic competition as a market structure together with the incorporation of fiscal policy in some manner. This is generally encompassed under ‘New Keynesian Economics’ and the ‘New Trade Theory’ which has tended to suggest the use of fiscal policy as a means to bringing the economy to a higher level of welfare and efficiency. Subsidy provisions also come under this policy instrument. We therefore give a quick review of some of the relevant academic literature for preliminary insights and intuition.

Solow (1998) provides an intuitive introduction to the effects monopolistic competition can bring to the macroeconomy. Paraphrasing, the existence of a monopolistically competitive equilibrium yields an outcome which he terms as a

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[9] This is mentioned and covered in Chapter 3 of Dixit (1990).
[10] See the references within Lancaster (1980).
[11] This approach was also critiqued with regards to the consistency of results obtained. See Hendler (1975).

case of ‘excess supply’ – i.e. firms are always willing to supply more output at the prevailing price. Bénassy (1987) formally models this situation incorporating a labour market and concludes the same. Hence, policies that raise total output and the supply of labour (for Bénassy’s case) will also lead to a rise in welfare.

One means by which this can be achieved is via the increase in aggregate demand which results from the government behaving as a consumer and purchasing the output of firms. Caraballo and Usabiaga (2006) survey some of this literature. The intuitive logic is as follows. An *ad hoc* increase in government purchases potentially raises the output and profits of firms. When the increased profits are redistributed as income to consumers, this creates higher demand from consumers owing to the higher income. A cycle which is akin to a Keynesian-type of multiplier effect thus perpetuates, ultimately giving a welfare improving outcome[12].

Alternatively, an increase in output is also possible if the government subsidises production costs as opposed to raising demand. This goes very much in line with the empirical observations mentioned earlier. Intuitively, by lowering firms’ production costs, each firm will be able to produce a larger quantity or more firms will enter the market, raising aggregate supply. Assuming a constant aggregate demand, prices have to fall in order for the market equilibrium to be maintained. And this can lead to a potentially welfare improving outcome. An early investigation on this comes from Negishi (1962). In his analysis, he shows the possibility of welfare gains as a result of subsidising new entrants into the market, so long as firms keep to the marginal cost pricing rule. Spence (1976b) and Dixit and Stiglitz (1977) also analyse the welfare outcomes of a provision of subsidies to firms and reach similar conclusions.

[12] This result may not be as clear-cut as we have mentioned here. Heijdra and van der Ploeg (1996) show that in the presence of free entry and exit of firms in the economy, the multiplier can be absent altogether.

Both forms of analysis can also potentially extend to the open economy context as illustrated by Matsuyama (1992) and Chandra *et al* (2002).

1.5. Closing Comments

We have established thus far that there is the theoretical possibility that the use of industrial policy in the form of subsidies can result in positive welfare gains. A preliminary overview of the theoretical literature suggests that this is indeed the case. However, evaluation of the empirical literature appears to indicate otherwise.

This seeming disconnection between theory and empirics provides us with a motivation to examine this in further detail. More importantly, there are strong indications that subsidies will continue to remain a significant part of government policy even if their proven effectiveness are mixed at best. The incentive ‘bidding’ packages for FDI documented by Charlton (2003) provides a strong case for this.

It should be stated here that we are neither advocating nor opposing the use of subsidies. What we are interested in examining is what specific outcomes can result from the use of this policy instrument, especially to welfare. More importantly, we want to determine if a policy rule which maximises welfare exists, before we can conclude on the desirability regarding the use of subsidies.

Chapter 2

Optimal Subsidies under Monopolistic Competition

2.1. Introduction

This chapter develops the benchmark model to illustrate the welfare effects of subsidising monopolistically competitive firms. We assume a closed economy and treat labour supply as exogenous and examine how different types of subsidies impact the level of welfare in the economy. We illustrate how the presence of monopolistic competition potentially validates supply-sided policy intervention by a benevolent government into the economy. We also derive the optimal subsidy rules that maximise welfare.

Monopolistic competition in the spirit of Chamberlin has provided rich and diverse insights both to the economic literature and toward policy analysis. Some features of this market structure are that firms produce differentiated products and have decreasing average costs, or increasing returns to scale, in production. Furthermore, each firm holds some degree of individual market power over its own product and, as such, will produce at a level away from the lowest point of their average cost curves, similar to a monopoly.

However, Chamberlin (1950) argues that as a result of a large number of differentiated product firms in the market, each firm does not engage in strategic behaviour with its rivals, while consumers value diversity in consumption. These are key elements of monopolistic competition which imply that there are no efficiency issues as those associated with a monopoly[1]. This is despite firms having production

[1] This stands in contrast against Joan Robinson who views monopolistic competition as yielding an

patterns similar to a monopoly. Furthermore, social welfare is actually optimised as a result of the preference for diversity[2].

Yet, Chamberlin does not actually dismiss the notion that some form of government intervention which changes the range and/or quantity of products available in the market could possibly be desirable. He concedes this to be valid so long as the welfare gain in doing so exceeds that lost from a reduction in variety or consumption. This was stated more succinctly by Norman (1989) who lists some objectives regarding socially beneficial policy options involving an industry subject to increasing returns to scale of production. These include increasing output of the industry, inducing new firms into the sector and raising the quantity of varieties available for consumption among others.

Developments in modelling monopolistic competition[3] after Chamberlin (1950) suggest that welfare *is* indeed lower under monopolistic competition as compared to the benchmark case of perfect competition. Spence (1976a) presents an early attempt in quantifying this welfare loss using a series of simulations. His results suggest that when firms exercise their monopoly power and diverge from marginal cost pricing, social welfare is lower owing to the higher prices it has to pay for consumption and the availability of a welfare-suboptimal number of varieties in the market. Dixit and Stiglitz (1977) confirm this sub-optimality and further show that society has an under-provision of variety and welfare is lower than in an economy under perfect competition.

To correct for this welfare loss, Dixit and Stiglitz (1977) proposed in their

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inefficient outcome. This non-concurrence was mentioned by Bellante (2004) who reiterates Chamberlin's assertion that monopolistic competition need not be non-Pareto optimal.

[2] Note that Chamberlin's normative analysis was qualitative in nature. As is now well-known, Dixit and Stiglitz (1977) have shown by means of a formal model that there is typically an under-provision of varieties and welfare is actually sub-optimal.

[3] Some of these 'developers' include Kelvin Lancaster, Michael Spence, and Dixit and Stiglitz. Dixit (2004) gives a brief overview of the models developed by each.

original contribution for all firms to receive a lump sum transfer which equals their individual fixed costs. All firms will thus price their output at marginal cost and new firms will be induced to enter the economy. Subsequently, social welfare is higher such that the economy reaches a first best optimum. Furthering along this line of thought, one can posit that if lump sum transfers have a welfare improving effect when there are no resource constraints, is there a possibility for the same to happen if there are limits to how much firms can receive, i.e. under a resource constraint?

To the best of our knowledge, this has yet to be studied in detail. Thus, the first objective of this chapter is to examine the use of a lump sum subsidy to firms in the increasing returns to scale sector when the government is subject to a constraint on its budget. Specifically, we are interested in determining if there is indeed an optimal lump sum transfer to firms that raises society's welfare, *given* the corresponding level of tax increases needed to finance it.

The second pertains to the pricing rule due to monopoly power. If firms hold some degree of market power over their product and behave like a monopolist in their pricing behaviour, would it be beneficial to subsidise production costs of each unit to allow firms to reduce their selling price? Coen (1951) explored this qualitatively and suggested that subsidies reducing both marginal and average costs and enlarging the scale of production can only benefit society. Taking this as our second point of departure, we examine this postulation and examine if there exists some optimal subsidy rule which lowers either the marginal or the average cost of production of firms in the increasing returns to scale sector.

The remainder of the chapter is structured as follows. In the rest of this section, we review briefly some of the literature pertaining to the application of monopolistic competition in macroeconomics; in particular we shall focus on whether it 'legitimises' public policy introduction. Section 2.2 lays out the basic Dixit and Stiglitz (1977) model which we will use for analysis. We examine the effects of the provision of subsidies and the resulting outcomes to welfare in Section 2.3 while Section 2.4 discusses briefly the intuition behind the obtained results. Section 2.5 concludes the chapter. The graphs referenced are collected in the accompanying appendix at the end of the chapter.

2.1.1. Monopolistic Competition and Macroeconomics

A lure of monopolistic competition for economic analysis stems from the view that it is a better description of the real world as compared to the perfectly competitive paradigm which underpinned most of previous analyses. Harrod (1967) credits Chamberlin for successfully reconciling textbook pedagogy with observations of the real world. With monopolistic competition, Chamberlin had also set a stage for future work and research to be built upon.

One area which benefited from the introduction of monopolistic competition is the field of Keynesian economics. As Naish (1993) mentions, an assumption of rigid wage levels was conventionally made in Keynesian economics as a reason for the inability of markets to adjust and clear in the Walrasian paradigm. While plausible, this assumption was often relaxed in later studies due to empirical observations that labour supply can even be infinitely elastic in the short-run. This runs contra to the effect of rigid wages which imply an inelastic labour supply. Conversely, monopolistic competition focuses attention on firms who act as price setters. This was perhaps an observation which appeared more consistent with the real world. Models of monopolistic competition such as that of Dixit and Stiglitz (1977) placed focus on the individual actions of each actor in the economy within a general equilibrium framework. This provided Keynesian economics with microeconomic grounding. In the words of Mankiw (1992), this meant that "Keynesian economics has been reincarnated into a body with firm microeconomic muscle."

A comprehensive coverage of what the economic literature now terms as the 'New Keynesian macroeconomics' is out of the purview of this chapter. The interested reader is referred instead to Bénassy (1995) who gives an intuitive and accessible overview to the micro-foundations and characteristics pertaining to this school of models. Alternatively, Silvestre (1993) provides a more detailed and thorough coverage of the same topic, but with a different model of monopolistic competition from that of Bénassy (1995)[4].

2.1.2. Policy Use in the Presence of Monopolistic Competition

Monopolistic competition also gives an apparent incentive for the introduction of some form of macroeconomic policy to the economy. Taking a cue from Startz (1995), resource allocation in the economy is Pareto optimal under perfect competition and there is no role for any governmental intervention. Monopolistic competition with its sub-optimal welfare outcomes clearly indicates a situation to the contrary. Thus, if one assumes there is a benevolent and omniscient government seeking to maximise social welfare, public policy which shifts society to a higher level of welfare is 'justifiable' to undertake.

Theoretically, Ng (1980) gives an illustration of the effectiveness of policy use by examining the outcome of a rise in money supply on the labour market in an imperfectly competitive economy[5]. Ng (1982) further shows that any change in

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[4] Macroeconomics was not the only field to benefit from monopolistic competition. International trade is another with the evolution of what is now termed as the 'New Trade Theory' beginning with Krugman (1979) alongside a parallel and effectively identical development by Dixit and Norman (1980).

[5] Strictly speaking, monopolistic competition alone does not invalidate money-neutrality. See Bénassy (1995) or Heijdra and van der Ploeg (2002) for an exposition. Ng (1980) is mentioned here as an example on how the deviation from perfect competition could potentially legitimise policy

marginal costs associated with a value added tax will elicit a corresponding response in output[6]. In an open economy context, Takahashi (2006) finds that unilateral subsidies yield welfare gains within a two-country model of trade with differentiated goods. Empirical evidence for subsidies such as that offered by Venables and Smith (1986) and Yamamura (1986) do suggest they are not uncommonly used. Their findings also suggest that there are benefits to be obtained in the macroeconomic context, such as in economic growth and the development of infant industries when firms are accorded some form of subsidies from the government.

There is thus reason to believe that subsidy policies can be welfare improving when the economy shows monopolistically competitive behaviour. However, as it is unlikely that any government has an unlimited budget on which to spend, it would be interesting to examine if there is some level of subsidy which yields a Pareto-improvement before societal welfare begins to regress (if at all), i.e. an optimal subsidy rule.

2.2. The Basic Model

The model used here is based on the Dixit and Stiglitz (1977) model of monopolistic competition. As mentioned previously, this is the most widely used model within the monopolistic competition literature. There are a variety of reasons for this, as discussed, but for our purposes, keeping to the Dixit and Stiglitz (1977) framework allows our results to have a direct comparability with their original analysis and postulations.

Specifically, we assume an economy with an exogenously endowed labour force,

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introduction.

[6] An increase in marginal costs raises prices and lowers output and vice versa. Ng (1982) further analysed this in the case of imposing a sales tax with similar results. There seems no reason to suggest *a priori* that any form of subsidy to production costs will not yield a set of similar outcomes.

L, with two sectors which we name as agriculture and manufacturing. Agriculture produces a homogenous commodity under perfectly competitive conditions (whose quantity is denoted by A). The manufacturing sector is composed of a range of firms producing a variety of differentiated products, the aggregate quantity which we denote as Y[7]. Firms in this sector experience increasing returns to scale in production and possess some degree of monopoly power over their own individual product. Labour is taken to be homogenous and mobile between the sectors with L_A and L_Y denoting the respective labour demands of the homogenous and the differentiated sectors. Labour mobility across sectors implies the same unique wage is paid in both sectors. Firms have free entry and exit into the economy. This serves to drive all firm profits to zero in the long run.

2.2.1. Consumers

As labour is homogenous, we can focus on an aggregate representative household (which supplies labour) and is assumed to derive utility from consumption of both goods. The household's preferences are described by a Cobb-Douglas type of utility function,

$$U = \left(\frac{Y}{\mu}\right)^{\mu}\left(\frac{A}{1-\mu}\right)^{1-\mu}, \tag{2.1}$$

where $0 < \mu < 1$. Labour is mobile between sectors, and this gives wage equalisation across sectors. The budget constraint of this representative household can thus be expressed as:

$$wL = P_Y Y + P_A A, \tag{2.2}$$

where w is the economy-wide wage rate, P_Y is the price index of the manufacturing good and P_A is the price of the agricultural goods. Thus, consumers allocate their

[7] We will use the terms 'manufacturing' and 'differentiated' interchangeably to mean the same.

budget between the two goods, Y and A at the first stage of budgeting. Optimisation of Eq. (2.1) subject to the budget constraint in Eq. (2.2) gives the demand functions for each type of good,

$$Y^D = \frac{\mu w L}{P_Y}, \tag{2.3}$$

$$A^D = \frac{(1-\mu) w L}{P_A}. \tag{2.4}$$

Total expenditure on each class of good is thus $P_Y Y = \mu w L$ and $P_A A = (1-\mu) w L$.

At the second stage, each consumer allocates his expenditure on each variety making up the composite of the differentiated good based on the price of each individual variety. Manufactures consist of a continuum of N varieties of imperfect substitutes aggregated by a CES sub-utility function,

$$Y = \left(\int_0^N y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{2.5}$$

where $\sigma > 1$ is the constant elasticity of substitution between varieties.

A characteristic of the Dixit-Stiglitz model is that consumption is divided among all available varieties with equal weights. Thus, should a subsequent change in the number of varieties occur, the expenditure share on each variety will be adjusted accordingly and symmetrically taking into account of this change. The consumer's objective therefore, is to maximise his consumption of manufactures subject to his expenditure on Y, with expenditure defined as:

$$P_Y Y = \int_0^N p_i y_i di. \tag{2.6}$$

Optimising Eq. (2.5) subject to Eq. (2.6) allows us to find the marginal rate of substitution between any two varieties and their relative price ratios, which we can use subsequently to derive the corresponding price index of manufactures[8], P_Y, as

$$P_Y = \left(\int_0^N p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \tag{2.7}$$

Notice that the price index is dependent on the number of varieties available for consumption. Strictly speaking, N is the mass of varieties available, or the extent of differentiation in the market, and not a number. However, for an easier intuitive grasp, we shall use ‘number’ to mean the same here-on. It can be seen from Eq. (2.7) that, *ceteris paribus*, as N increases, P_Y falls. With the price index, we can subsequently obtain the demand function of a typical variety i as

$$y_i^D = \frac{\mu w L}{P_Y} \left(\frac{p_i}{P_Y} \right)^{-\sigma}. \tag{2.8}$$

It is also straightforward to derive an expression for the aggregate consumer’s indirect utility by substituting the demand functions for each class of good into Eq. (2.1), yielding:

$$V = \frac{wL}{P_Y^{\mu} P_A^{1-\mu}}, \tag{2.9}$$

where the denominator indicates the ‘cost-of-living’ or simply general price index.

From Eq. (2.9), it is evident that utility is inversely proportional to the cost-of-living index when the expression of P_Y is substituted into V. As P_Y is dependent on the number of varieties N, this implies that all else held constant, an individual will attain a higher level of utility as N increases. Dixit-Stiglitz models are thus also termed as ‘love-of-variety’ models.

2.2.2. Firms

On the supply side, firms in the manufacturing sector are profit-maximising entities

[8] See Chapter 13 of Heijdra and van der Ploeg (2002) for a derivation of the price index.

and each produces a variety available for consumption. Each firm is a sole producer of its output and prices it independently of its competitors in accordance to the behaviour of a profit-maximising monopolist, taking the industry level price index as given. This behaviour is justified by virtue of the fact that the total number of firms in the model is assumed to be large. Specifically, the number of firms operating in the industry is determined endogenously via free entry and exit into the market up till where each firm makes zero profits in equilibrium.

As firms are assumed to be symmetric[9], we can drop the subscript i denoting an individual firm for notational convenience. Firms use only labour as the sole factor input and each firm faces the following cost function in production,

$$TC = wl_Y, \tag{2.10}$$

where l_Y is the total labour demanded by the firm and w, the wage rate paid to labour. The objective of each firm is to maximise its profit,

$$\pi = py^D - wl_Y, \tag{2.11}$$

subject to demand, where y^D is the total demand for the firm's product.

Each firm uses an increasing returns to scale technology for production which requires a fixed input of α units of labour for production to take place. An input of β units of labour is needed for each unit of output, such that total labour demand per firm is

$$l_Y = \alpha + \beta y. \tag{2.12}$$

Substituting Eq. (2.12) into Eq. (2.10), we can obtain the marginal cost of each unit of output or

$$MC = \frac{\partial TC}{\partial y} = \beta w. \tag{2.13}$$

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[9] This is akin to the single representative firm modelling framework in Ng (1980, 1982). Aggregate output of the manufacturing sector is thus the total output of all firms in the sector or $Y = Ny$.

Furthermore, it is straightforward to see that each firm experiences a falling average cost of production[10], giving rise to an incentive for each firm to specialise, with each firm producing only one variety of output. This implies that with N varieties available for consumption in the market, the total number of firms operating in the sector is also N. Maximising each firm's profit subject to the demand for its own variety, the price each firm charges is found to be

$$p = \frac{\sigma}{\sigma - 1}\beta w. \tag{2.14}$$

The expression for p shows that the optimal pricing formula is a constant mark-up of $\frac{\sigma}{\sigma-1}$ over the firm's marginal cost of each unit of output in Eq. (2.13).

Following Chamberlin's arguments, monopolistic competition is characterised by the presence of a large number of firms in the market. There are no market barriers and firms have free entry and exit into the market. Firms will enter the market in the presence of positive profits or conversely, exit when they incur losses. Free entry and exit thus leads to all profits being competed away up to the point where each firm in the market makes zero profits. Thus, using Eq. (2.12) and Eq. (2.14) into Eq. (2.11) and imposing the zero-profit condition, we find the equilibrium output of each firm as

$$y^S = \frac{\alpha(\sigma - 1)}{\beta}. \tag{2.15}$$

Eq. (2.15) implies that a firm's optimal output scale is constant and independent of the number of competitors it faces. This standard result stems from the constant elasticity of substitution between varieties and from the absence of strategic interaction between firms under monopolistic competition.

[10] The average cost, which we denote as C, of a unit is $C = \frac{(\alpha+\beta y)w}{y}$. Differentiating with respect to y gives $\frac{\partial C}{\partial y} = \frac{-\alpha w}{y^2}$. Average cost is thus falling with every unit increase in output.

With the equilibrium price and quantity defined, we can now find the number of firms that exist in equilibrium. Assuming full employment and given the assumed symmetry between firms, total labour demand from the increasing returns sector is found as

$$L_Y = Nl_{Yi}. \tag{2.16}$$

Using Eq. (2.12) and Eq. (2.15) into Eq. (2.16), the number of firms, *N*, sustainable by the market is thus found to be:

$$N = \frac{L_Y}{\alpha\sigma}. \tag{2.17}$$

Eq. (2.17) has the added implication that the number of firms is dependent on the proportion of labour in manufacturing. A change in L_Y, *ceteris paribus*, will similarly result in a change in the number of firms in this sector.

The agricultural good is produced and sold under perfect competition. This also implies that it is priced at marginal cost. The profit function of a typical firm in this sector takes the form

$$\pi_A = P_A A^S - wL_A. \tag{2.18}$$

Assuming that the production of *A* is produced under constant returns to scale with a unit labour requirement of unity, we rewrite

$$L_A = A^S; \tag{2.19}$$

imposing the zero profit condition, it follows that $P_A = w$. The assumed inter-sectoral labour mobility gives rise to wage equalisation across sectors with $w = P_A$ being also the prevailing wage rate in the manufacturing sector.

To close the model, the total employment of labour in the two sectors needs to equal the economy's labour endowment, i.e. $L = L_Y + L_A$.

2.2.3. Normalisations and General Equilibrium

With agriculture assumed to be perfectly competitive, it is convenient to use it as the numeraire good, and normalise its price to unity: $P_A = 1$. It therefore follows that the wage rate in equilibrium is found to be $w = 1$. Given the symmetry assumption, we can then rewrite Eq. (2.7) as:

$$P_Y = \left(Np^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

which, making use of Eq. (2.14), can be written as

$$P_Y = N^{\frac{1}{1-\sigma}}\left(\frac{\sigma\beta}{\sigma-1}\right). \tag{2.20}$$

Using Eq. (2.20), $P_A = 1$ and $w = 1$ into Eq. (2.9), the indirect utility function can now be written as

$$V = \frac{L}{\left(N^{\frac{1}{1-\sigma}}\left(\frac{\sigma\beta}{\sigma-1}\right)\right)^{\mu}}, \tag{2.21}$$

The indirect utility function given by Eq. (2.21) re-illustrates the point we made earlier regarding the 'love-of-variety' property of Eq. (2.9) in greater clarity. With the normalisations, the primary driver of the representative consumer's utility level is now P_Y, the price level of manufactured goods. As noted, from Eq. (2.20), it can be seen that P_Y is a decreasing function of N, the number of varieties. As N increases, the general price index as given in the denominator falls correspondingly, i.e. welfare increases as the number of varieties increases: given consumers 'love variety', other things equal, their utility increases as more varieties are available for consumption.

Using Eq. (2.8), Eq. (2.14), Eq. (2.15) together with Eq. (2.20), we can solve for N in terms of all exogenous variables to obtain: $N = \frac{\mu L}{\alpha\sigma}$. The allocation of labour to each sector of the economy can be obtained using Eq. (2.16) together with the labour market clearing condition. The resulting labour allocation to the manufacturing and the agricultural sectors are found to be $L_Y = \mu L$ and $L_A = (1-\mu)L$ respectively.

2.3. Subsidy Provisions

The subsidies we consider here consist of two types. The first relates back to the postulation in the seminal contribution of Dixit and Stiglitz (1977). They suggest that in order for a firm to practice marginal costs pricing, the government should provide lump sum transfers which covers the losses that will be incurred arising from fixed costs. The second corresponds to subsidies which affect unit production costs as mentioned by Coen (1951). For this subsidy type, we will examine the provision of subsidies which lowers the marginal cost and the average cost of production of each unit of output.

An interesting question to consider first is how the subsidy should be financed. Tax theory suggests that if an economy is modelled using a representative individual or household, the economy is simply an aggregation of all individuals such that the optimal type of tax to impose is a lump sum tax[11]. In reality, a more common means for governments to raise revenue is by levying a tax rate on some portion of income instead [12]: Salanie (2003), for example, suggests that revenue obtained from proportional income taxation makes up as much as a quarter of total government budgets among OECD countries. This particular financing scheme was also adopted by Takahashi (2006) in his analysis and we follow suit here[13] .

[11] See Mankiw *et al* (2009).
[12] Furthermore, as earnings do differ according to individuals, the tax schedule typically consists of different tax rates for different income levels, e.g. in a progressive fashion.
[13] Using a proportional tax system raises questions about distortions to welfare and labour supply. Salanie (2003) provides a textbook treatment in Chapter 2. Note that as we assumed an exogenous labour supply, the tax does not affect the supply of labour.

Assuming therefore, that the government prescribes a value of the subsidy and allows the tax rate to vary to balance its budget, what the subsidy does is alter the cost and profit functions of the receiving firms in the manufacturing sector. This intuitively suggests that the subsidy gives firms supernormal profits in the immediate aftermath. In the long run however, free entry and exit into the sector will change the number of firms that exist up to where each firm once again makes zero profits.

2.3.1. Demand Effects

With the imposition of the tax, the demand functions of the representative consumer will be affected given the reduction in income. Denoting the tax variable as τ and bearing in mind that $w = P_A = 1$, the budget constraint becomes $(1-\tau)L = P_Y Y + A$. Maximising utility subject to the new budget constraint gives the new demand functions for manufactures and the agricultural good respectively as

$$y_\tau^D = \frac{\mu L(1-\tau)}{P_Y}\left(\frac{p}{P_Y}\right)^{-\sigma}, \tag{2.22}$$

and

$$A_\tau^D = (1-\mu)(1-\tau)L. \tag{2.23}$$

The corresponding indirect utility function is now

$$V_\tau = \frac{(1-\tau)L}{P_Y^\mu} \tag{2.24}$$

Combining the production function of the A-sector with Eq. (2.23), the new labour demand in agriculture can also be obtained to be

$$L_A = (1-\mu)(1-\tau)L\,. \tag{2.25}$$

Recall that total labour endowment in the economy is also the sum of labour demand in both sectors, $L = L_A + L_Y$. For the labour market to clear therefore, labour demand in (and its allocation to) the manufacturing sector must satisfy the labour constraint, i.e.

$$L_Y = L - (1-\mu)(1-\tau)L \tag{2.26}$$

2.3.2. Lump Sum Transfers

Practical use of lump sum subsidies is probably most commonly found when governments offer a set amount to firms to entice them to set up production in a certain location. These can be thought of, for instance, as being in terms of a low-cost land lease, joint ventures undertaken by the government together with the firm, or even simply just a lump sum tax exemption or tax rebate. Its one-off payment nature also makes them comparatively easy to implement.

We assume that each firm receives a subsidy, s, where $0 < s \leq \alpha$ [14]. Denoting post-subsidy variables with an s subscript, this augments each firm's total cost and profit functions to

$$TC_s = \alpha + \beta y_s^S - s, \tag{2.27}$$

and

$$\pi_s = p y_\tau^D - (\alpha + \beta y_s^S) + s. \tag{2.28}$$

Differentiating the total cost functions will show that under such a subsidy, each firm's marginal cost remains constant at $MC = \beta$, with the profit-maximising price unchanged as $\frac{\sigma\beta}{\sigma-1}$. However, what is affected is each firm's fixed costs – and hence its optimal output scale. The latter is obtained by imposing the zero-profit condition:

$$y_s^S = \frac{(\alpha - s)(\sigma - 1)}{\beta} \tag{2.29}$$

[14] The case of $s = \alpha$ is the subsidy in the unconstrained optimum examined by Dixit and Stiglitz (1977).

and is clearly lower than the equilibrium output per firm in the unsubsidised equilibrium. The intuition for this is straightforward. As the government subsidises firms, their fixed cost fall and hence profits increase. However, increased profits will lead to entry of new firms up to the point where normal profits are once again reached. In equilibrium, each firm thus becomes smaller, i.e. the optimal output scale falls. The market clearing condition for the manufactured goods sector is

$$\frac{(1-\tau)\mu L}{P_Y}\left(\frac{p}{P_Y}\right)^{-\sigma} = \frac{(\alpha - s)(\sigma - 1)}{\beta}, \tag{2.30}$$

with the homogeneous sector being

$$(1-\tau)(1-\mu)L = A_s^S. \tag{2.31}$$

The labour market equilibrium is given by

$$(1-\tau)(1-\mu)L + N_s l_Y = L$$

Which, upon substituting for l_Y and y_s^S, is written as

$$(1-\tau)(1-\mu)L + N_s\left((\alpha - s)\sigma + s\right) = L. \tag{2.32}$$

The first term reflects the labour demand in the agricultural sector, while the second is the total labour requirement by firms in the differentiated goods sector. The government's budget constraint is written as

$$\tau L = N_s s. \tag{2.33}$$

Eq. (2.33) says that the total tax revenue must equal the subsidy bill, i.e. the subsidy rate multiplied by the number of firms in equilibrium. From the labour market equilibrium and government budget constraint, we can analyse the outcomes from the subsidy qualitatively.

Taking s to be exogenous (i.e. assuming that the government fixes it *ad hoc*), we solve N_s in terms of τ and s from the labour market equilibrium condition and the government budget constraint. The corresponding solutions to (2.32) and (2.33) are

$$N_s = \frac{(\mu + (1-\mu)\tau)L}{(\alpha - s)\sigma + s}$$

and

$$N_s = \frac{\tau L}{s}$$

respectively. These two equilibrium conditions are plotted for given values of σ, μ, L and $s < \alpha$ in Figure 2-1 in the N and τ space[15]. *LME* denotes the labour market equilibrium while the government budget constraint is labelled *GBC*. These loci show the combinations of N and τ where the labour market is in equilibrium and the government budget constraint balances – for any given *ad hoc* value of subsidy. The equilibrium value of N_s and τ is found where the two curves intersect, i.e. at a point such as E. We also examine the result of an increase in the subsidy level. The new equilibrium loci are depicted with dotted lines and the new equilibrium points are denoted with a prime.

The intuition behind the slopes of these lines is straightforward. The *LME* is upward sloping because at any point on the line, an increase in the number of firms will raise labour demand. The vertical arrows indicate the direction in which N will move above and below the *LME*. The *GBC* is also upward sloping as an increase in the number of firms for any particular level of subsidy will mean the government now has a higher subsidy bill. This increased subsidy bill will need to be balanced by an increase in taxes. The horizontal arrows show the adjustment paths that τ will take above and below the *GBC*.

The analysis of the effects of a rise in the level of subsidy can be conducted by

[15] Note that the solutions for N_s are not actually linear, but expressing them diagrammatically as straight lines makes analysis easier.

examining how the two lines shift. Starting from the solid lines in Figure 2-1, suppose that the government increases s. This leads to an increase in the number of firms which means labour demand in the sector increases, such that the *LME* shifts upwards. The subsidy bill is also now higher such that the government has to raise taxes. This results in the *GBC* moving rightwards. From the arrow configurations which indicate the adjustment paths, we can see there is a convergence towards a new stable equilibrium with the solutions indicated by E'.

Therefore, solving for the two endogenous variables, N_s and τ, for any value of s that the government chooses, from Eq. (2.32) and Eq. (2.33) gives

$$N_s = \frac{\mu L}{(\alpha - s)\sigma + \mu s}, \tag{2.34}$$

$$\tau = \frac{\mu s}{(\alpha - s)\sigma + \mu s}. \tag{2.35}$$

The solution for N_s as given in Eq. (2.34) validates the initial conclusions obtained from Figure 2-1. That is, as s increases from $0 \rightarrow \alpha$, the denominator becomes smaller and N_s correspondingly increases. Figure 2-2 illustrates the tax-subsidy schedule and shows clearly that a higher level of subsidy meets with a greater than proportional increase in the corresponding tax rate needed to finance it. Inserting *ad hoc* values of s such that s increases linearly into τ, we find that τ increases at an increasing rate. Using $s = 0.25\alpha$, the tax rate is found as $\tau_{s=0.25\alpha} = \frac{\mu}{\mu+3\sigma}$. Taking a linear increase in s using $s = 0.5\alpha$ and $s = \alpha$ yield the tax rates of $\tau_{s=0.5\alpha} = \frac{\mu}{\mu+\sigma}$ and $\tau_{s=\alpha} = 1$ respectively.

An increasing subsidy implies that higher tax revenues are required to finance it. This leads to a fall in disposable income and a corresponding reduction in the demand for each firm’s product. Thus, to induce more firms to enter, the government has to provide larger lump sum transfers which need to be financed by higher taxes. In the polar case where $s \to \alpha$, the tax rate approaches 1, and the consumer actually has no disposable income. This clearly questions the feasibility of lump sum transfers equalling α.

While $s = \alpha$ is a doubtful prospect, the shape of Figure 2-2 also hints at the existence of a welfare maximising subsidy rule. To find the optimal policy therefore, we have to obtain values of τ and s, such that they maximise society's indirect utility function given by Eq. (2.24). Substituting the relevant expressions for P_Y, p, N_s and τ into V_τ, this becomes

$$V_\tau = \frac{\left(1 - \frac{\mu s}{(\alpha - s)\sigma + \mu s}\right)L}{\left(\left(\frac{\mu L}{(\alpha - s)\sigma + \mu s}\right)^{\frac{1}{1-\sigma}} \frac{\sigma\beta}{\sigma - 1}\right)^{\mu}}. \tag{2.36}$$

Differentiating with respect to s gives

$$\frac{\partial V_\tau}{\partial s} = \frac{\mu\sigma L(\alpha - \alpha\mu - s\sigma + \mu s)}{(\sigma - 1)((\alpha - s)\sigma + \mu s)^2}\left(\left(\frac{\mu L}{(\alpha - s)\sigma + \mu s}\right)^{\frac{1}{1-\sigma}} \frac{\sigma\beta}{\sigma - 1}\right)^{-\mu} \tag{2.37}$$

which we equate to zero and solve for s. This yields the optimal subsidy, or s^*, to be

$$s^* = \frac{\alpha(1 - \mu)}{\sigma - \mu}, \tag{2.38}$$

which is simply a fraction of fixed costs, α. Substituting s^* into N_s and τ gives the new equilibrium number of firms and the tax rate that are consistent with the optimal subsidy in the general equilibrium. These are found as

$$N_s^* = \frac{\mu L}{\alpha(\sigma - 1 + \mu)} \quad \text{and} \tag{2.39}$$

$$\tau^* = \frac{\mu(1 - \mu)}{(\sigma - \mu)(\sigma - 1 + \mu)}. \tag{2.40}$$

Note that while the number of varieties has increased post-subsidy, it should be remembered that there is also a fall in output per variety as shown from Eq. (2.29). The former leads to gains in welfare while the latter works in the opposite direction. Since the subsidy level is set to maximise V_τ, we need to confirm that s^* does indeed yield a Pareto improvement. Differentiating the utility function twice and substituting s^* for s, we obtain

$$\left.\frac{\partial^2 V_\tau}{\partial s^2}\right|_{s=s^*} = \frac{\mu\sigma L(\mu-\sigma)}{\alpha^2(\sigma-1)(\sigma-1+\mu)^2}\left(\left(\frac{\mu L}{\alpha(\sigma-1+\mu)}\right)^{\frac{1}{1-\sigma}}\frac{\sigma\beta}{\sigma-1}\right)^{-\mu} < 0. \tag{2.41}$$

This verifies that the solution does yield a new higher optima and welfare gains are indeed a possibility from lump sum transfers to firms at a fraction, $\frac{1-\mu}{\sigma-\mu}$, of total fixed costs.

2.3.3. Per Unit Payments

Per unit subsidies paid towards the costs of each additional unit of output are generally done to lower the costs of production of each unit of output, giving the firm an incentive to increase production. One form which this subsidy instrument can take is a sales rebate by the government to the firm. This and other equivalent forms of subsidy provisions are discussed briefly in Yamamura (1986) to which the reader is referred to.

We assume here that the government gives a production subsidy, s, which covers a part of the marginal cost, β, of production of every unit of output the firm produces. As such, the cost and profit functions are modified as follows:

$$TC_s = \alpha + \beta y_s^S - s y_s^S, \tag{2.42}$$

$$\pi_s = p y_\tau^D - \left(\alpha + \beta y_s^S\right) + s y_s^S. \tag{2.43}$$

where $0 < s < \beta$ [16]. Differentiating Eq. (2.42) with respect to *s*, the marginal cost of production of each output is now found to be

$$MC_s = \beta - s \tag{2.44}$$

and the new profit-maximising price becomes

$$p_s = \frac{\sigma(\beta - s)}{\sigma - 1}. \tag{2.45}$$

Using Eq. (2.45) into Eq. (2.44) and imposing the zero-profit condition, each firm's optimal output is now higher with

$$y_s^S = \frac{\alpha(\sigma - 1)}{\beta - s} \tag{2.46}$$

This runs in tandem with the analysis of Ng (1982). It also hints that there are potential welfare gains coming from higher quantities of the manufacturing good available for consumption.

Together with Eq. (2.22), Eq. (2.23) and Eq. (2.26), the equilibrium conditions for both types of goods as well as the labour market are defined as

$$(1-\tau)(1-\mu)L = A_s^S, \tag{2.47}$$

$$\frac{(1-\tau)\mu L}{P_Y}\left(\frac{p}{P_Y}\right)^{-\sigma} = \frac{\alpha(\sigma - 1)}{\beta - s}, \tag{2.48}$$

and

$$(1-\tau)(1-\mu)L + N_s \frac{\alpha(\sigma\beta - s)}{\beta - s} = L \tag{2.49}$$

respectively. Unlike the lump sum subsidy, the amount of subsidy which now needs to be financed depends on both the number of firms existing in equilibrium and the

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[16] Unlike the lump sum subsidy case, we restrict the upper bound for s to be less than β. While it is theoretically possible for the subsidy to equal the marginal cost of production, following Coen (1951), we assume that the government's motive in providing the subsidy is to raise the production scale of the firm. If we allow for $s = \beta$, it follows that $MC = 0$ and we will get $y = \infty$ which is unrealistic. Placing the upper bound limit prevents this outcome.

output of each firm. Given that the government pays the subsidy to each unit produced, the budget constraint in this case is

$$\tau L = N_s s y_s^S = N_s s \frac{\alpha(\sigma - 1)}{\beta - s} \tag{2.50}$$

As before, we use the *LME*, given by Eq. (2.49), and *GBC*, given by Eq. (2.50), for a preliminary qualitative analysis as well as to check for stability. Taking s as exogenous and solving for N_s and τ, these are illustrated in Figure 2-3. The intuition behind the lines is identical as before. We similarly examine the effect of an increase in the subsidy rate shown by the broken lines. An interesting observation here is that while a stable equilibrium exists, the number of firms falls when the subsidy increases. This is unlike the lump sum case and also runs counter to our preliminary intuition which suggests that subsidies would increase firm numbers instead. Using the two equilibrium conditions, we can obtain the solutions for N_s and τ for any *ad hoc* value of s, yielding

$$N_s = \frac{\mu L(\beta - s)}{\alpha(\sigma\beta - s(\mu + \sigma(1 - \mu)))} \tag{2.51}$$

$$\tau = \frac{\mu s(\sigma - 1)}{\sigma\beta - s(\mu + \sigma(1 - \mu))} \tag{2.52}$$

Examining Eq. (2.51), it is easy to see that as $s \to \beta$, $N_s \to 0$. Thus, while a subsidy for every unit of output results in a rise in the quantity of each variety produced, it also leads to a reduction in the number of varieties available for consumption instead.

The tax rate schedule follows a similar shape to that in Figure 2-2. In this case however, with a linear increase in the subsidy of $s = 0.25\beta$, $s = 0.5\beta$ and $s = \beta$, the corresponding tax rates are $\tau_{s=0.25\beta} = \frac{\mu(\sigma-1)}{\mu(\sigma-1)+3\sigma}$, $\tau_{s=0.5\beta} = \frac{\mu(\sigma-1)}{\mu(\sigma-1)+\sigma}$ and $\tau_{s=\beta} = 1$. The values of τ also indicate that our earlier restriction on the subsidy is reasonable, as setting a subsidy level of $s=\beta$ simply leaves the consumer with no expenditure income.

To obtain the optimal policy rule, we substitute Eq. (2.51) and Eq. (2.52) into the indirect utility function to obtain:

$$V_\tau = \frac{\left(1-\frac{\mu s(\sigma-1)}{\sigma\beta - s(\mu+\sigma(1-\mu))}\right)L}{\left(\left(\frac{\mu L(\beta-s)}{\alpha(\sigma\beta - s(\mu+\sigma(1-\mu)))}\right)^{\frac{1}{1-\sigma}}\frac{\sigma(\beta-s)}{\sigma-1}\right)^{\mu}}. \tag{2.53}$$

Differentiating with respect to s gives

$$\frac{\partial V_\tau}{\partial s} = \frac{\mu\sigma L(\beta(1-\mu)-s(\mu+\sigma(1-\mu)))}{(\sigma-1)(\sigma\beta - s(\mu+\sigma(1-\mu)))^2}$$
$$\left(\left(\frac{\mu L(\beta-s)}{\alpha(\sigma\beta - s(\mu+\sigma(1-\mu)))}\right)^{\frac{1}{1-\sigma}}\frac{\sigma(\beta-s)}{\sigma-1}\right)^{-\mu}. \tag{2.54}$$

Setting Eq. (2.54) equal to zero and solving for s, we get

$$s^* = \frac{\beta(1-\mu)}{\mu+\sigma(1-\mu)}. \tag{2.55}$$

Similar to the lump sum handout case, the optimal subsidy is a fraction of the subsidised cost (the variable cost β in this case). In turn, number of firms and tax rate that correspond to the optimal subsidy are

$$N_s^* = \frac{\mu L(\sigma(1-\mu)+2\mu-1)}{\alpha(\sigma-1+\mu)(\mu+\sigma(1-\mu))} \tag{2.56}$$

and

$$\tau^* = \frac{\mu(1-\mu)(\sigma-1)}{(\mu+\sigma(1-\mu))(\sigma-1+\mu)} \tag{2.57}$$

respectively.

With this subsidy, the optimal number of firms is smaller than that of the unsubsidised equilibrium. This results from the increased output per firm. As each firm produces more, demand is constrained by the size of the market, L, and total disposable income. The larger equilibrium output per firm then implies that a smaller number of firms will be sustained by the market in equilibrium. Given consumers love variety and that variety has fallen, we have to verify that this policy rule does indeed bring utility to a higher optimum. Therefore, taking the second derivative of V_τ and substituting s^* for s, we get

$$\left.\frac{\partial^2 V_\tau}{\partial s^2}\right|_{s=s^*} = \left(\left(\frac{\mu L(\sigma(1-\mu)+2\mu-1)}{\alpha(\sigma-1+\mu)(\sigma(1-\mu)+\mu)} \right)^{\frac{1}{1-\sigma}} \frac{\sigma\beta(\sigma(1-\mu)+2\mu-1)}{(\sigma-1)(\sigma(1-\mu)+\mu)} \right)^{-\mu}$$

$$\frac{\mu\sigma L(\sigma(\mu-1)-\mu)}{\beta^2(\sigma-1+\mu)^2} < 0. \qquad (2.58)$$

Hence, with both the positive optimal subsidy value and the result that $\frac{\partial^2 V_\tau}{\partial s^2} < 0$, this implies that a welfare improvement is indeed realised. Instead of increased variety, the driver in this case is from the resulting increase in consumption per variety that stems from the fall of the price of each manufacturing variety. This more than compensates for the reduction in available varieties resulting from the lower number of firms which exist in the post-subsidy equilibrium.

2.3.4. Subsidising Average Costs (or Wages)

We turn now to consider subsidising average costs. As labour is the sole production input, this subsidy is in effect a wage subsidy paid to firms for every unit of labour employed. With a subsidy of $0 < s < 1$, which covers a part of wages, the cost and profit functions are now changed to

$$TC_s = (\alpha + \beta y_s^S)(1-s), \qquad (2.59)$$

and

$$\pi_s = py_\tau^D - \left(\alpha + \beta y_s^S\right)(1-s). \tag{2.60}$$

The marginal cost of each unit's production then becomes

$$MC_s = \frac{\partial TC_s}{\partial y_s^S} = \beta(1-s), \tag{2.61}$$

with the new profit-maximising price set at

$$p_s = \frac{\sigma\beta(1-s)}{\sigma - 1}. \tag{2.62}$$

It is straightforward to verify that this type of subsidy does not alter the representative firm's equilibrium size: Using Eq. (2.62) into Eq. (2.60), imposing the zero-profit condition and solving for y gives the optimal output scale, which remains as per the unsubsidised equilibrium given by Eq. (2.15).

Therefore, with demand as given by Eq. (2.22), the market clearing condition of a typical variety is

$$\frac{(1-\tau)\mu L}{P_Y}\left(\frac{p}{P_Y}\right)^{-\sigma} = \frac{\alpha(\sigma-1)}{\beta} \tag{2.63}$$

with the market clearing for agriculture remaining as Eq. (2.23). The labour market is now defined by the equilibrium condition:

$$(1-\tau)(1-\mu)L + N_s l_Y = L.$$

which simplifies to:

$$N_s \alpha\sigma = \left(\mu + (1-\mu)\tau\right)L\,. \tag{2.64}$$

The government budget constraint is given by:

$$\tau L = N_s l_Y s. \tag{2.65}$$

As the subsidy is paid to firms for each unit of labour employed, substituting for l_Y gives $\tau L = N_s \alpha\sigma s$. The qualitative analysis of equilibrium and stability is again done using the *LME* and the *GBC* curves that are depicted in Figure 2-4 together with a subsequent increase in s shown by the dotted lines. Note a caveat here with respect to the *LME*. From Eq. (2.64), the labour market equilibrium solves to give

$$N_s = \frac{(\mu + (1-\mu)\tau)L}{\alpha\sigma}, \tag{2.66}$$

Hence, unlike the lump sum and per unit subsidies case, the *LME* is now independent of the subsidy provided by the government. Any change in the equilibrium number of firms now results in a movement along the *LME* instead of a shifting of the locus as in the previous two cases. A general observation that can be made from Figure 2-4 is that even though the *LME* does not shift, the total number of firms will be higher with the provision of a higher subsidy level. Thus, while firm scale remains unchanged from the unsubsidised equilibrium, the reduction in average costs of production resulting from the subsidy will result in new entry of firms into the sector.

As before, taking the subsidy to be fixed *ad hoc*, we solve for the general equilibrium solutions for N_s and τ to give

$$N_s = \frac{\mu L}{\alpha\sigma(1 - s(1-\mu))} \tag{2.67}$$

$$\tau = \frac{\mu s}{1 - s(1-\mu)} \tag{2.68}$$

A quick examination of Eq. (2.67) confirms the observation regarding firm numbers. As $s \rightarrow 1$, $N_s \rightarrow \infty$. The tax rate exhibits the same incremental path given in Figure 2-2, with $\tau_{s=0.25} = \frac{\mu}{3+\mu}$, $\tau_{s=0.5} = \frac{\mu}{1+\mu}$ and $\tau_{s=1} = 1$.

Similar to the previous cases, there ought also to be some level of tax-subsidy combination here which maximises welfare. Inserting Eq. (2.67) and Eq. (2.68) into V_τ,we obtain:

$$V_{\tau} = \frac{\left(1 - \frac{\mu s}{1 - s(1-\mu)}\right)L}{\left(\left(\frac{\mu L}{\alpha\sigma(1 - s(1-\mu))}\right)^{\frac{1}{1-\sigma}} \frac{\sigma\beta(1-s)}{\sigma - 1}\right)^{\mu}}. \tag{2.69}$$

Differentiating it with respect to *s* gives:

$$\frac{\partial V_{\tau}}{\partial s} = \frac{\mu L(1 - s\sigma)(1-\mu)}{(\sigma - 1)(1 - s(1-\mu))^2}\left(\left(\frac{\mu L}{\alpha\sigma(1 - s(1-\mu))}\right)^{\frac{1}{1-\sigma}} \frac{\sigma\beta(1-s)}{\sigma - 1}\right)^{-\mu} \tag{2.70}$$

Equating to zero and solving for *s* to obtain the optimal policy rule, we find

$$s^{*} = \frac{1}{\sigma}. \tag{2.71}$$

The optimal subsidy rule in this case is extremely straightforward when compared with the previous cases. The value of the optimal subsidy is simply the inverse of the elasticity of substitution between varieties. For example, if we assume an elasticity of substitution of $\sigma = 5$ between each variety, the subsidy is 20% of the average cost of a unit of output. This analysis suggests that the informational requirement of this type of policy for the government may be lower than in previous cases as there is no requirement for knowledge of either the fixed or the variable cost of production for this subsidy to be implemented.

Substituting for *s*, the corresponding firm number and tax rate to the subsidy are

$$N_{s}^{*} = \frac{\mu L}{\alpha(\sigma - 1 + \mu)} \tag{2.72}$$

$$\tau^{*} = \frac{\mu}{\sigma - 1 + \mu}. \tag{2.73}$$

To confirm that the policy attains a maxima rather than a minimum point, we get:

$$\left.\frac{\partial^2 V_\tau}{\partial s^2}\right|_{s=s^*} = \left(\left(\frac{\mu L}{\alpha(\sigma-1+\mu)}\right)^{\frac{1}{1-\sigma}}\beta\right)^{-\mu} \frac{\mu\sigma^3 L(\mu-1)}{(\sigma-1)(\sigma-1+\mu)^2} < 0, \tag{2.74}$$

that is, a Pareto improvement is indeed the outcome of this subsidisation policy.

2.4. Discussion

The general conclusion we can make thus far is that subsidies to firms exhibiting increasing returns to scale do raise social welfare. Apart from the second derivatives from Eq. (2.41), Eq. (2.58) and Eq. (2.74) all being $\left.\frac{\partial^2 V_\tau}{\partial s^2}\right|_{s=s^*} < 0$, an alternative means to verify this is by examining the change in V_τ for a very small change in subsidy provision. Substituting $s = 0$ into Eq. (2.37), Eq. (2.54) and Eq. (2.70), it can be checked that $\left.\frac{\partial V_\tau}{\partial s}\right|_{s=0} > 0$ in all cases, indicating there are welfare gains when a subsidy is introduced. The existence of a positive tax-subsidy policy rule in all cases thus provides some degree of legitimacy for governments in introducing industry-focused subsidy policies.

An interesting observation to first make here is on the original postulation of the seminal contribution of Dixit and Stiglitz (1977). They suggest that a government could potentially provide lump sum transfers to firms for them to cover their fixed costs. Firms can subsequently price their output at marginal cost, resulting in the textbook case of marginal cost pricing as in perfect competition. We find instead that such a policy does not yield an optimal outcome. Rather, when firms hold a certain degree of monopoly power (as determined by σ), the elimination of fixed costs does not yield an optimal welfare outcome. So long as firms exercise their monopoly power and price their output at a mark-up over marginal cost, marginal cost pricing will only be reached in the limiting case of $\sigma \to \infty$. However, in such a situation, the economy will comprise solely of the homogeneous sector[17].

The solutions of s^* and τ^* also make for some interesting commentary. The most obvious observation is that the elasticity of substitution between varieties, σ, features prominently in all the expressions for s^* and τ^*. In all cases, as $\sigma \to 1$, manufacturing varieties are more differentiated and this leads to larger tax and subsidy values. The converse holds when $\sigma \to \infty$. An intuitive way to view this is that when σ is small, the additional utility a consumer derives from the consumption of an additional variety exceeds by far the utility lost from a reduction in quantity consumed of all the other existing varieties. Thus, he is willing to give up more disposable income (via an increase in taxation) so long as there are larger numbers of varieties subsequently available. The reverse holds with a larger value of σ.

From the firms' perspective, the elasticity of substitution between varieties in the *Y*-sector can be interpreted as an indication of the degree of specialisation inherent in the production of each firm's product. Highly differentiated products, as denoted by a small value of σ could suggest that firms in essence belong to different industry types, where inter-firm production technologies and/or techniques are largely incompatible. Hence, there is a need for larger subsidies to induce entry or to increase the scale of production in the *Y*-sector as a whole. Conversely, when σ is large, it is akin to considering that firms are all within a similar industry group with relatively homogenous products with similar production techniques and technologies. A small subsidy is then sufficient to entice firms to either increase output or enter the market.

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[17] When $\sigma \to \infty$, it follows on that $p \to \beta$ and $s \to 0$. However, each firm in the differentiated sector now incurs a loss of α needed for production to commence and will not meet the zero-profit condition. All firms in the differentiated sector will subsequently exit, leaving just the homogeneous sector in existence.

Also, while a subsidy is indeed welfare improving, it should be noted that the mechanism involved differs in the three cases. Recall that consumers love variety as reflected by the role the price index, P_Y, plays in determining utility. P_Y is dependent on the number of varieties, N, and a larger N lowers P_Y, raising welfare. In the case of lump sum transfers, this welfare gain is driven wholly by the increase in N. The average cost subsidy achieves this by a combination of an increased N as well as a lower p. The production subsidy engineers a reduction in the number of varieties in the market. Instead, the reduction in price from the subsidy dominates the loss in variety to the extent that P_Y is reduced sufficiently to still yield an increase in utility.

The other observation worth mentioning here comes from the solutions in Eq. (2.38), Eq. (2.55) and Eq. (2.71). In Eq. (2.38) and Eq. (2.55), it is clear that apart from σ, s^* also depends on the relevant cost component (respectively α and β) and the expenditure share, μ. This is unlike Eq. (2.71) where the optimal subsidy simply depends on σ. Whilst it is intuitive as to why α and β enter the optimal subsidy rules, the role of μ in s^* is less clear.

We can possibly consider the following. The parameter μ determines the size of the Y-sector by virtue of the expenditure share of income, or μI. When either α or β is subsidised, the subsidy reduces the optimal firm scale but raises N in the former, and vice versa in the latter. μ becomes a necessary consideration in determining the optimal subsidy as it serves to 'constrain' the size of N or y from going to the unrealistic $N = \infty$ and $y = \infty$. This however, does not occur for the case of the average cost subsidy as the subsidy does not affect individual firm scale. Also, in the polar case of $s \rightarrow 1$, the size of N remains bounded by L[18], i.e. there are no possible extreme results. The role of the subsidy in this case, is to reduce the monopoly

[18] It can be checked from Eq. (2.67) that when $s = 1$, $N = \frac{L}{\alpha\sigma}$.

power of each individual firm, and bridge the existing wedge between price and marginal costs. This depends only on σ. Thus, μ does not enter into the optimal subsidy rule[19].

From the indirect utility functions of Eq. (2.36), Eq. (2.53) and Eq. (2.69), it is evident that the level of welfare depends on the structural parameters α, β, μ, σ, L and s. However, the only variable which differs in each case is s. Substituting the respective s^* into each indirect utility function, labelling the lump sum, per unit and average cost subsidies with the subscripts of *LS*, *PU* and *AC* respectively, we can obtain a welfare ranking between the three policy regimes. The indirect utility function for the lump sum subsidy becomes:

$$V_{LS} = \frac{\left(\frac{\sigma(\sigma-1)}{\sigma(\sigma-1)+\mu(1-\mu)}\right)L}{\left(\left(\frac{\mu L(\sigma-\mu)}{\alpha(\sigma(\sigma-1)+\mu(1-\mu))}\right)^{\frac{1}{1-\sigma}}\frac{\sigma\beta}{\sigma-1}\right)^{\mu}}.$$

which is obtained by substitution of Eq (2.38) into Eq. (2.36).

Upon substitution of Eq. (2.55) into Eq. (2.53) yields the level of welfare associated with the per unit subsidy, that is:

$$V_{PU} = \frac{\frac{\sigma L}{(\sigma-1+\mu)}\left(\frac{\sigma(1-\mu)+\mu}{\sigma(1-\mu)-1+2\mu}\right)^{\frac{\sigma(1-\mu)-1+2\mu}{1-\sigma}}}{\left(\left(\frac{\mu L}{\alpha(\sigma-1+\mu)}\right)^{\frac{1}{1-\sigma}}\frac{\sigma\beta}{\sigma-1}\right)^{\mu}}.$$

Finally, by substituting Eq. (2.71) into Eq. (2.69) we obtain the indirect utility function for the average cost subsidy:

[19] It is possible that μ has other roles than just as a constraint. However, that is beyond the objective and focus of this chapter and it can be an interesting avenue for future work.

$$V_{AC} = \frac{\left(\frac{\sigma-1}{\sigma-1+\mu}\right)L}{\left(\left(\frac{\mu L}{\alpha(\sigma-1+\mu)}\right)^{\frac{1}{1-\sigma}}\beta\right)^{\mu}}$$

It is easy to verify that comparison of these three levels of welfare yields the following ranking:

$V_{AC} > V_{PU} > V_{LS}$.

For a government aiming to attain the highest level of social welfare, it is thus clear that the average cost subsidy policy would be the one to implement.

Note however, that this does not imply the superiority of one subsidy over the other. As mentioned, all the subsidy regimes yield a Pareto improvement via a different transmission mechanism. Our assessment is based on the assumption of a benevolent government who uses the level of social welfare as its decision criterion. On this alone, the average cost subsidy yields the highest level of utility and is in that sense, the ‘ideal’ policy to use.

2.5. Concluding Comments

We began this chapter by considering the words of Chamberlin (1950) and Norman (1989) regarding the ‘inefficiency’ of monopolistic competition and the aims that public policy ought to take under such a market structure. We then focused specifically on the postulations of both Coen (1951) as well as those of Dixit and Stiglitz (1977) who suggest for the provision of subsidies to firms as a policy instrument to correct for this inherent inefficiency in monopolistic competition. Our primary results verify the proposition that subsidies are welfare improving when firms experience increasing returns to scale in production and the economy exhibits monopolistically competitive behaviour.

In this chapter, we considered the use of a tax-subsidy policy combination where a welfare-maximising government imposes a proportional tax on income to finance a subsidy given to firms in the monopolistically competitive sector. We derive and examine the optimal subsidies for three types of subsidies: a lump sum transfer to firms, a subsidy for every unit of output produced, and a subsidy to wages. The subsequent solutions for the optimal subsidy, s^*, show that the welfare-maximising subsidy is a proportion of fixed costs, marginal costs and wages respectively. Thus, a benevolent government aiming to maximise social welfare appears well-justified to implement a similar tax-subsidy policy combination as we have used here.

A practical relevance of our results suggests that subsidies are viable when used for targeting the objectives stated by Norman (1989). Depending on the subsidy variant used, subsidies do indeed fulfil the objectives of increasing the number of varieties, inducing entry of new firms or firms having higher output levels as a result. More importantly, monopolistic competition suggests that the resulting social outcome is not Pareto optimal. Introducing policy tools which remove some degree of this sub-optimality and allowing society to move to higher welfare levels is therefore potentially desirable. On the basis of our findings, subsidies appear a potential and viable policy instrument to realise this objective.

Appendix

Figure 2-1: Quasi-Phase Diagram of Equilibrium with a Lump Sum Subsidy

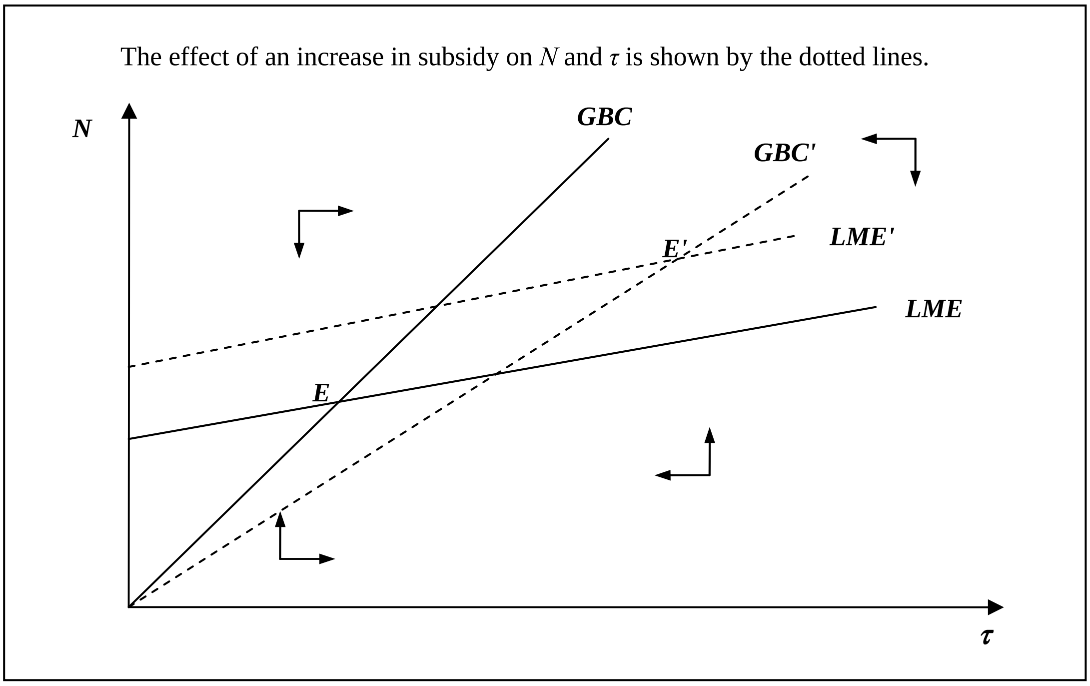


Figure 2-2: Tax-Subsidy Schedule with *ad hoc* values of *s*

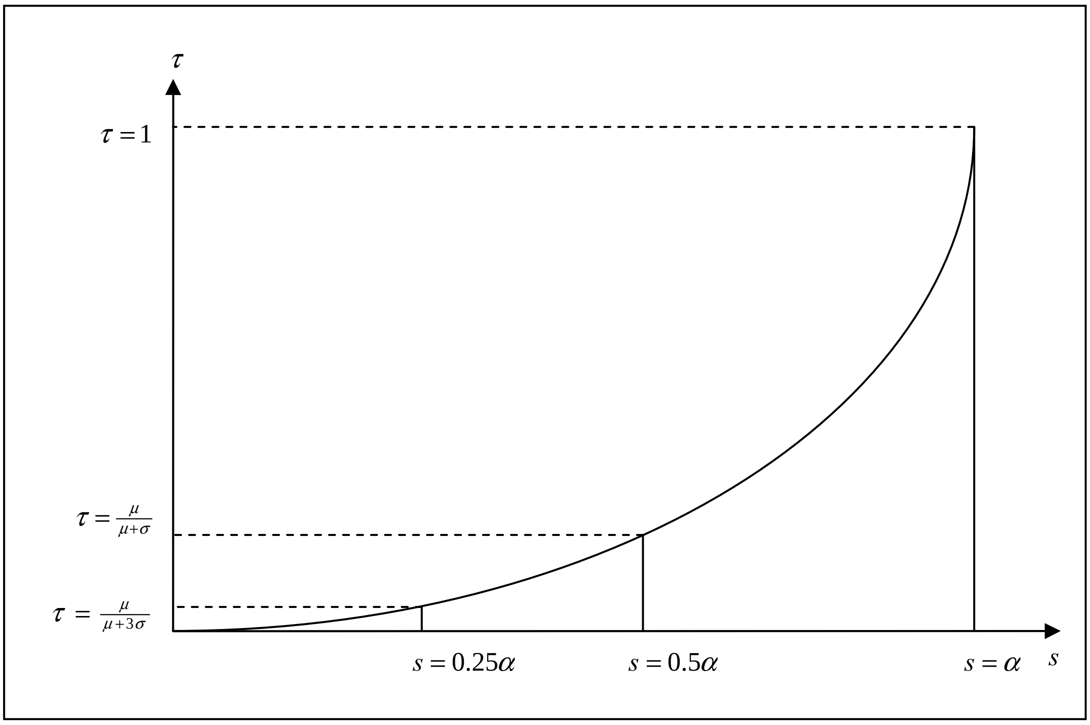


Figure 2-3: Quasi-Phase Diagram of Equilibrium with a Per Unit Subsidy

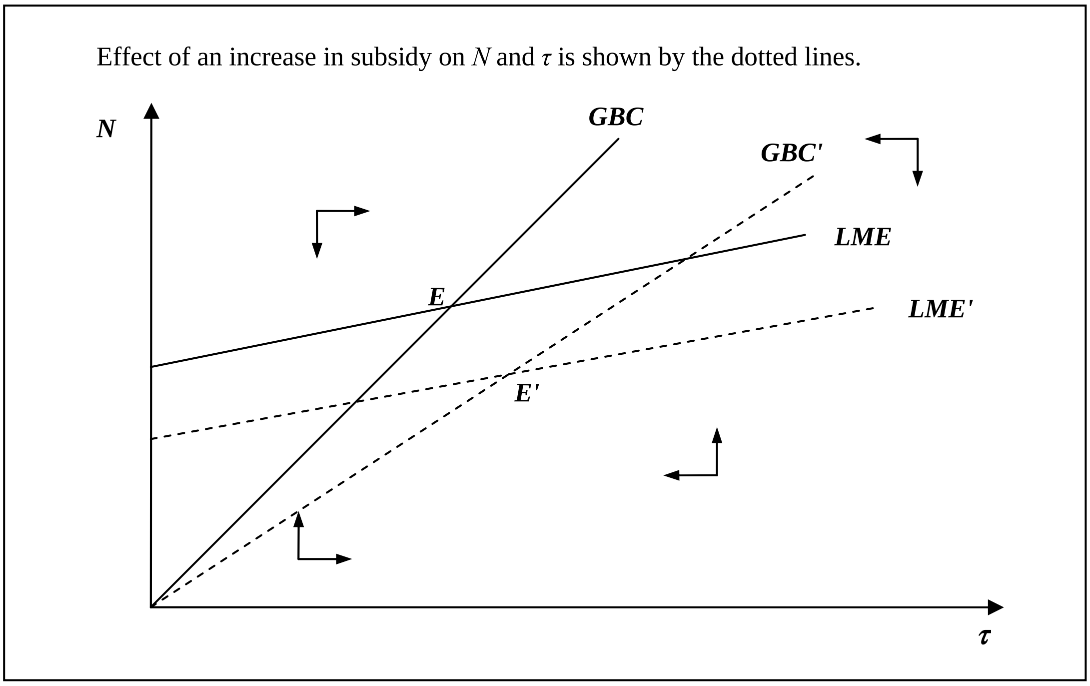
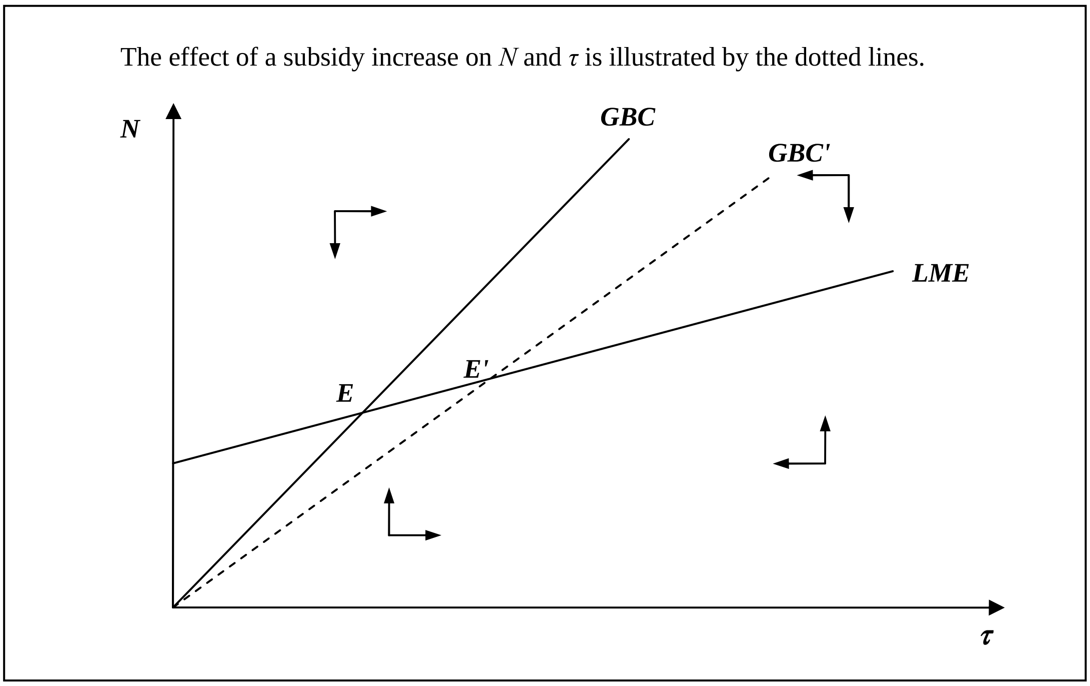


Figure 2-4: Quasi-Phase Diagram of Equilibrium with an Average Cost Subsidy



Chapter 3

Subsidies and the Effect on Aggregate Activity

3.1. Introduction

This chapter expands on the analysis in Chapter 2 by incorporating an endogenously determined labour supply into the benchmark model and examining how subsidies affect welfare and the labour supply. Allowing for labour supply to vary can potentially yield insights for policy deliberations, especially in the realm of regional and labour market policies. As a starting point of our analysis, we use the model presented in the seminal contribution of Blanchard and Kiyotaki (1987) combined with some elements of the model in Heijdra *et al* (1998).

New Keynesian macroeconomic models incorporating increasing returns to scale in production and individual market power of firms are known to give rise to outcomes that are inefficient and non-welfare maximising. When product markets are referred to as monopolistically competitive, the final good produced by these firms are characterised to be an imperfect substitute with the next and each individual firm holds a degree of market power for their output. Even in a setting where firms have free entry and exit into the market, the market power of each firm is not diminished and each continues to act as if it were a monopoly producer of its own product.

Following Spence (1976b) and Dixit and Stiglitz (1977), social welfare in these models is determined by both the quantity and the number of varieties of differentiated goods that are available for consumption. Yet, when firms are bound to a certain level of fixed costs in production which gives rise to internal increasing returns to scale, free entry and exit of firms subject to a zero-profit condition result in the provision of a number of varieties which is less than the first-best outcome[1]. This bears the further implication that aggregate output and welfare in the economy will also be lower. Hence, social welfare can be raised from either an increase in output or the number of varieties or both. This was subsequently analysed and validated by Corchón (1991).

Apart from welfare considerations, a secondary implication of monopolistic competition is that under-utilisation of resources may occur. Assuming labour to be the only factor of production under a textbook case of perfect competition, the wage paid to a worker equals his marginal product of labour. However, firms sell their output at a mark-up over marginal costs under monopolistic competition. This mark-up implies that a firm's marginal revenue product is greater than the marginal product of labour. Subsequently, this divergence results in an under-utilisation of resources, i.e. un- or under-employment, as workers remain paid at their marginal product. Bénassy (1993) gives a detailed exposition of this outcome[2].

3.2. Aim and Layout

If an underutilisation of productive resources exists in the economy, we can hypothesise that there may be welfare gains to be realised if the level of economic activity can be raised. A higher level of economic activity would, *ceteris paribus*, increase both output and the number of varieties available for consumption, thereby raising welfare. Intuitively, if some government policy results in a higher level of economic activity, there will result an increase in aggregate output and a reduction in

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[1] See the analysis by Dixit and Stiglitz (1977) and the survey by Lancaster (1990).

[2] Matheron (2002) and Matheron and Maury (2004) present some simulation estimates on the welfare costs of consumption arising from monopolistically competitive firms via an endogenous growth model with both capital and labour. While their focus is different from ours, they do provide a quantitative backing to the contention of welfare sub-optimality under monopolistic competition.

the amount of idle resources in the economy. Ultimately, social welfare will be higher as a whole as a result of the introduction of the policy.

Thus, the focus of this chapter is on how influencing the level of economic activity affects welfare using subsidies[3] as the policy instrument. Specifically, we want to examine how subsidies affect the level of activity in the economy, together with the corresponding effects they generate. We define the level of economic activity as the quantity of labour supplied in the economy – which is endogenously determined. This is done by allowing a representative individual to choose his optimal supply of labour. We shall study the re-allocative effects (if any) that emerge from the policy and also derive the optimal policy rule for a welfare-maximising government. We find that there is indeed an optimal subsidy level which results in a Pareto improvement. Also, our other results presented here can potentially carry some non-trivial weight and relevance for policy considerations, some of which we will briefly address later.

The rest of the chapter is structured as follows. In the next section, we review some of the relevant background literature pertinent to our analysis. Section 3.4 lays out the basic model used for the subsequent analysis. Section 3.5 examines the effect of three different types of subsidies on welfare and the level of economic activity and derives the optimal policy rules. Section 3.6 discusses briefly the policy implications the results could offer. Section 3.7 concludes the chapter. The appendix at the end contains a mathematical appendix and all the graphs referred to within the chapter.

[3] We use subsidies in the generic sense rather than referring to any specific form such as R&D or preferential tax rates – even though the different types of subsidy discussed below lend themselves to different interpretations.

3.3. Background Literature

When imperfect competition is incorporated into macroeconomics, the directions of study are not limited to welfare sub-optimality or the possible presence of excess resource capacity. However, additional factors and implications such as menu costs for instance, are beyond the immediate scope of this chapter and a concise and intuitive approach can be obtained from Solow (1998, Chapter 1).

Most relevant to us here are the seminal contributions of Hart (1982), Rowe (1987), and Blanchard and Kiyotaki (1987) who incorporate market power of firms into their analyses[4]. They come to the following similar set of conclusions on the impact of monopolistically competitive firms in the broader economy as a whole. Firstly, the level of economic activity is too low in an imperfectly competitive market and welfare is suboptimal when compared to the benchmark of perfect competition. Secondly, some form of expenditure by the government over and above what individual consumers expend can influence the level of aggregate output in the economy. This in turn, translates to possible welfare gains[5].

The prospect regarding the possible role of a government in influencing the economy has given rise to a burgeoning amount of interest and resulting literature. However, it is necessary to first note here that models which examine the effect of government expenditure (and fiscal policy, more generally) make up one of the existing strands of literature within the framework of New Keynesian macroeconomic models. Another group of models are concerned with the analysis of

[4] Both Hart (1982) and Blanchard and Kiyotaki (1987) incorporate some degree of specialisation to labour, but Rowe (1987) does not incorporate an explicit labour market into his analysis.

[5] Monopolistic competition often assumes that economic agents do not take into account the effect of their actions on others. This could possibly be somewhat unconvincing to some. Weinrich (2007) expands on Blanchard and Kiyotaki (1987) by incorporating the assumption that agents *do* consider their individual impact on others. He finds welfare losses to be even higher than what was calculated by Blanchard and Kiyotaki (1987). This further strengthens the cause for the introduction of some type of government policy to correct for this welfare sub-optimality.

the impact that monetary policy will bring with respect to output. Our concern and focal point of interest here lies with the former and readers who are interested in the second can refer to the surveys and references within Rotemberg (1987) and Clarida *et al* (1999) who cover the features of this latter group of models.

3.3.1. Consumption-related Expenditure

Starting from the seminal work of Hart (1982) and Blanchard and Kiyotaki (1987), the primary implication of their analyses is to lend support to the plausibility of a welfare-maximising government having the means to raise welfare via the use of public sector spending. Such expenditure would raise the overall level of activity and aggregate output in the economy, with the stimulus for expansion coming via a Keynesian-type multiplier effect. This would, in effect, provide a possible rationale for governments to use a form of tax-and-spend policy[6]. Rowe (1987) however, finds the actual outcome of public spending to be indeterminate and suggests that it would be premature to draw conclusions on how fiscal policy could potentially influence the economy as a whole.

The emphasis of existing analyses in this area has been focused on the degree by which government expenditure affects the overall aggregate level of activity in the economy. An autonomous increase in government spending triggers an adjustment process which results in a more than proportionate increase in output in the economy, consistent with a Keynesian-type multiplier effect. Some examples in the literature along this strand include Mankiw (1988), Startz (1989), Dixon and Lawler (1996), Solow (1998, Chapter 3) and Devereux *et al* (2000)[7]. Most of their results appear to

[6] To finance the expenditure, the government would have to levy a tax on firms and/or consumers to raise funds. This in turn however, would result in distortionary effects which may run counter to any potential welfare gains depending on the mode of taxation used. See Auerbach and Hines (2002) who provide a comprehensive review of these issues.

[7] For a more substantial review of the theory and of its ramifications, we refer to the surveys by

support the hypothesis that increases in government expenditure are welfare improving.

However, this conclusion is far from definite. Dixon and Lawler (1996), in particular, caution against any attempt to generalise existing results for policy purposes. They point out that many of the results obtained are strongly dependent upon particular or specific types of assumptions and/or on the modelling framework adopted. In particular, they point out that the presence of a multiplier is very much dependent on factors such as whether there is free entry and exit of firms in the economy, the time horizon, i.e. short- or long-run, type of government spending or even the type of utility function used. Should assumptions concerning one or more of these be altered, they can potentially change the overall outcome.

This lack of robustness was demonstrated by Reinhorn (1998) who derives an interestingly counter-intuitive result by extending Mankiw's (1988) model. Mankiw (1988) assumes the government uses debt financing, or that taxes remain unchanged following a fiscal expansion. Reinhorn (1998) however, explicitly imposes a balanced budget condition for the government and finds instead, that the optimal fiscal policy to be one of no government expenditure at all. Furthermore, Heijdra *et al* (1998) find a crowding out effect of government expenditure using distortionary taxation together with free entry and exit of firms into the economy. They show that when government expenditure increases, there is the possibility of a negative multiplier, or simply that a greater amount of public spending results in a contraction of national income as a result of increased government spending.

Hemming *et al* (2002) review both the theoretical and empirical literature concerning the effectiveness and size of fiscal multipliers. Their broad conclusions

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Bénassy (1993) and Dixon and Rankin (1994).

are that the size of the fiscal multiplier can, in theory, be both positive or negative in magnitude depending on assumptions such as if Ricardian equivalence holds, or whether or not competition exists between public and private consumption. The empirical evidence however, suggests that multipliers are generally positive, but smaller in magnitude than what might be theorised. Their conclusions simply further the level of ambiguity on the actual welfare benefits of government expenditure policies.

3.3.2. Cost-related Expenditure

As an alternative to direct government spending which affects demand for goods and services, fiscal policy can also be used to influence the output of firms, or by reallocating productive resources between different agents and sectors in the economy. Such policies may possibly (and, in some instances, more appropriately) fall under the auspices of the term 'industrial policy' where the expenditure is targeted towards reducing production-related costs[8] rather than towards increasing final demand to effect changes in output levels. In discussions of industrial policy making and outcomes, such aims have even been explicitly stated, such as for the case of Japan (Sato, 1990), and France (Cohen, 2007).

Empirical case studies for Northern Ireland (Roper and O'Shea, 1991), East Germany (Begg and Portes, 1993), Japan (Nolan, 1993), and Finland (Kangasharju, 2007) seem to suggest that such policy aims can be implemented with potentially successful and beneficial outcomes[9]. Such conclusions lend support to the assertion

[8] This can come in the form of cost subsidies, tax incentives and preferential loan treatments. See Sato (1990) for a more in-depth review.

[9] The indicators used in these papers as a welfare measurement are, typically, changes in income, unemployment and output levels. Barring Nolan (1993), the broad conclusion one seems to see is that industrial policies raise income and output and lower unemployment. This could indirectly be implied to mean welfare gains. However, Nolan (1993) suggests that while Japan evolved in her industrial output and trade towards higher value-added sectors as a result of industrial policy, the supposed

that policies targeted at stimulating employment creation and development of specific industrial sectors or even simply towards increasing total output are indeed effective. More essentially, an underlying implication one might be tempted to draw is that government spending in such a manner is socially beneficial, similar to the consumption-type of expenditure much of the existing theoretical literature is interested in analysing.

However, the theoretical literature relating to the use and effects of cost-reducing subsidy policies appear to be somewhat comparatively less developed. In their original analysis, Dixit and Stiglitz (1977) bring up the proposition that a government provides lump sum subsidies in order that firms can cover their level of fixed costs. This will in turn, raise the number of firms (and hence varieties of the good) and overall output, thereby increasing social welfare. Developing along this thread, Costrell (1990) shows that under monopolistic competition, a subsidy to firms' production costs will always improve welfare, regardless of whether the varieties available are too many or too few, as the ultimate purpose of the subsidy is to reallocate production inputs to the differentiated goods sector.

However, rather than subsidising firms directly, Fleurbaey (1998) illustrates the prospect of employment subsidies as offering a means to raise the overall level of employment, which can increase both the level of economic activity and welfare. More recent contributions by Ng and Zhang (2007) and Jodoh (2008) also put forward the proposition of using production subsidies as a vehicle to raise the level of output, activity and welfare in the economy. The key driver behind this sort of effect is that the presence of monopolistic competition does not exhaust the productive capacity of the economy. Thus, changing of production costs through the

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welfare gains from such policy is potentially ambiguous, but not completely absent.

use of policy can potentially improve the utilisation of resources, leading to greater output and higher welfare levels.

While it has been established that there are welfare benefits from subsidising firms in the increasing returns to scale sector, the seminal contributions briefly reviewed thus far in relation to the use of subsidies have focused largely on the use of a subsidy to per unit production costs (e.g. Ng and Zhang, 2007, and Johdo, 2008). Besides the per unit production subsidy, we further complement the existing literature by examining the use of a subsidy targeted to average costs and fixed costs. We also add on to the existing literature by attempting to determine if there is some policy rule whereby welfare is optimised and study the various effects that subsidy policies can elicit in further detail.

The starting point for our analysis builds on the seminal contribution of Blanchard and Kiyotaki (1987). However, our model differs from theirs in that we assume that consumers have preferences that express love-of-variety. As a result, as in Heijdra *et al* (1998), the number of varieties available for consumption affects the level of welfare. We then allow for free entry and exit of firms into the increasing returns to scale sector, with the government using proportional income taxation as its subsidy financing instrument. The endogeneity of market structure allows us to account for how the policy intervention (i.e. the tax-policy mix) affects welfare via changes in the number of varieties, the labour supply, the aggregate output of the economy and the labour allocation among sectors[10].

[10] More ‘conventional’ New Keynesian macroeconomic models such as Mankiw (1988) and Startz (1989) normalise the love-of-variety parameter in the utility function to 1. In their analyses, consumers care only about absolute consumption. However, as Heijdra *et al* (1998) have illustrated, the preference for variety clearly has an impact on the size of the multiplier and welfare. We thus also incorporate love-of-variety into our model. The other main differentiating aspect is in the use of proportional income taxation as mentioned.

3.4. The Basic Model

We use a simplified version of the model by Blanchard and Kiyotaki (1987), where the labour market is competitive but labour supply is endogenous. Specifically, all consumers are identical, and they derive utility from the consumption of goods – with preferences that reflect love-of-variety (LOV) – and leisure[11]. Therefore, a representative consumer has a utility function defined as:

$$U = \left(\frac{Y}{\mu}\right)^{\mu}\left(\frac{A}{1-\mu}\right)^{1-\mu} - \frac{L^{1+\gamma}}{1+\gamma}, \tag{3.1}$$

where Y is an aggregate basket of a mass of differentiated products, A is a homogeneous good, and L is the labour supply of the consumer. The first term of the utility function represents the utility from consumption, where μ and $(1-\mu)$ are the consumption shares of each good. The second term reflects the disutility obtained from the supply of labour, with $\gamma \geq 0$ determining the elasticity of labour supply.

This type of utility function which removes the income effect on leisure is commonly used in the literature due to its interesting analytical advantages (mainly giving rise to a labour supply function that resembles those used in conventional macroeconomic textbook models). As Bénassy (1993) argues, when labour supply is elastic, there exists an under-utilisation of labour and consumers change their labour supply merely in response to a change in real wage. This utility function simply stylises this idea by eliminating the income effect and ruling out a 'backward bending' labour supply function.[12]

Income is derived solely from labour wages in this model and fully expended on consumption. The budget constraint can be written as $wL^S = P_Y Y + P_A A$ where L^S

[11] In Blanchard and Kiyotaki (1987), utility is obtained from goods, real money balances and leisure, with money used as the numeraire.
[12] Mankiw (1988) and Startz (1989) use a Cobb-Douglas utility function; for a review of the effect of incorporating the income effects of leisure, see Caraballo and Usabiaga (2006).

refers to the quantity of labour supplied.

The consumer faces a two-stage constrained utility maximisation problem. In the first stage, the consumer chooses the quantity of labour to supply and the consumption of each good given his overall budget constraint. Maximising Eq. (3.1) subject to the budget constraint given above yields the labour supply and the demand functions of each type of good, that is:

$$L^S = \left(\frac{w}{P_Y^{\mu} P_A^{1-\mu}} \right)^{\frac{1}{\gamma}}, \tag{3.2}$$

$$Y^D = \frac{\mu w L^S}{P_Y}, \tag{3.3}$$

and

$$A^D = \frac{(1-\mu) w L^S}{P_A}. \tag{3.4}$$

Eq. (3.2) indicates that the quantity of labour supplied is an increasing function of the real wage.

Consumer preferences of the differentiated good are defined over a continuum of varieties according to a CES aggregator as in Dixit and Stiglitz (1977), that is:

$$Y = \left(\int_0^N y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{3.5}$$

where N is the total mass[13] of varieties available for consumption and $\sigma > 1$ is the elasticity of substitution between varieties. The price index of the differentiated good is defined as

$$P_Y = \left(\int_0^N p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \tag{3.6}$$

[13] The mass of varieties available is often loosely referred to as the 'number' of varieties within the academic literature. We follow suit here by taking the two as synonymous.

Hence, in the second stage, the consumer maximises the CES sub-utility in Eq. (3.5) subject to the total expenditure on this good, given by $P_Y Y = \left(\int_0^N p_i y_i di\right)$, to obtain the demand for each variety, y_i:

$$y_i^D = \frac{\mu w L^S}{P_Y}\left(\frac{p_i}{P_Y}\right)^{-\sigma}. \tag{3.7}$$

Substituting the demand functions of Y^D and A^D with the labour supply function of Eq. (3.2) into the utility function, we obtain the indirect utility function which gives us a measure of aggregate welfare[14]:

$$V = \frac{\gamma}{1+\gamma}\left(\frac{w}{P_Y^{\mu} P_A^{1-\mu}}\right)^{\frac{1+\gamma}{\gamma}}. \tag{3.8}$$

It is clear from Eq. (3.8) that utility depends both on the level of real wages and on the elasticity of labour supply.

On the supply side, firms producing manufactures use an increasing returns to scale technology with labour as the sole input factor. Firms are symmetric, sharing the same technology characterised by a fixed labour requirement of α and a variable input, β. The assumption of symmetry between firms allows us to drop the subscript, i, used to denote each individual firm. The total labour requirement of a firm then takes the form:

$$l_Y = \alpha + \beta y^S. \tag{3.9}$$

It follows that the total cost and profit functions of a typical firm are respectively given by:

$$C = \left(\alpha + \beta y^S\right)w, \tag{3.10}$$

$$\pi = p y^D - \left(\alpha + \beta y^S\right)w. \tag{3.11}$$

With the Chamberlinian large group assumption, the elasticity of demand for

[14] The derivation of Eq. (3.8) is shown in Appendix A.

each product is approximated by σ. Maximising profits subject to demand implies that each firm's product is priced at a mark-up over its marginal cost, that is:

$$p = \frac{\sigma\beta w}{\sigma - 1}. \tag{3.12}$$

In equilibrium, free entry and exit of firms into the sector will eliminate supernormal profits. Therefore, substituting the price in Eq. (3.12) into the profit function in Eq. (3.11) and imposing the zero-profit condition, we obtain the equilibrium output of each firm as

$$y^S = \frac{\alpha(\sigma - 1)}{\beta}. \tag{3.13}$$

It is clear from Eq. (3.13) that the equilibrium optimal output scale for each firm is constant and does not depend on either the size of the market or the number of firms in the industry. This result stems from the absence of strategic interaction between firms and the constant elasticity of substitution assumption.

The number of firms that exist in equilibrium can now be obtained using the optimal firm scale in Eq. (3.13) together with the demand functions in Eq. (3.7) to obtain the market clearing condition for each variety, i.e. $y^D = y^S$. This gives

$$\left(\frac{p}{P_Y}\right)^{-\sigma} \frac{\mu w L^S}{P_Y} = \frac{\alpha(\sigma - 1)}{\beta}. \tag{3.14}$$

Substituting p and P_Y from Eq. (3.12) and Eq. (3.6) into Eq. (3.14), the number of firms that exist in equilibrium is found to be[15]:

$$N = \frac{\mu L^S}{\alpha\sigma}. \tag{3.15}$$

which, given Eq. (3.2), can be written as $N = \frac{\mu}{\alpha\sigma}\left(\frac{w}{P_Y^{\mu} P_A^{1-\mu}}\right)^{\frac{1}{\gamma}}$. Unlike the case where

[15] Note that as a result of the symmetry assumption, the integral in Eq. (3.6) can be rewritten as N multiplied a typical firm's variable or $P_Y = \left(Np^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$.

labour is inelastically supplied, as in the previous chapter, other things equal, the number of firms decreases in γ. Intuitively, *ceteris paribus*, a larger γ implies that labour supply becomes more inelastic, or that consumers value leisure more. Thus the quantity of labour supplied by each consumer decreases as γ increases, leading to a fall in total labour supply. This will subsequently lead to a, *ceteris paribus*, fall in the number of firms existing in equilibrium.

Turning to the other sector, the homogeneous good is produced and sold under perfectly competitive conditions. This also implies that it is priced at marginal cost. We shall use A as the numeraire good. The profit function of a typical firm in the homogeneous sector takes the form:

$$\pi_A = P_A A^S - w_A L_A. \tag{3.16}$$

Assuming that the production of A entails a unit labour requirement of one, i.e.:

$$L_A = A^S, \tag{3.17}$$

it follows that $P_A = w_A$. Normalising P_A to unity implies that that the agricultural wage rate is also equal to unity, i.e. $w_A = 1$. Inter-sectoral labour mobility ensures wage equalisation across sectors, hence the wage rate in the manufacturing sector is also equal to one, i.e. the economy-wide wage rate will be $w = 1$.

Using $w = 1$ together with L^S from Eq. (3.2) into N and evaluating, the number of firms in equilibrium is:

$$N = \left(\left(\frac{\mu}{\alpha\sigma} \right)^{\gamma} \left(\frac{\sigma - 1}{\sigma\beta} \right)^{\mu} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}}. \tag{3.18}$$

The aggregate level of economic activity in the economy can be measured in terms of the total quantity of labour employed across both sectors, defined as:

$$L^S = L_Y + L_A, \tag{3.19}$$

with $L_Y = Nl_Y$ and $L_A = A^S$ being the labour demand of the differentiated and homogeneous sectors respectively.

3.5. Subsidies and the Level of Aggregate Economic Activity

We now study the effect subsidies have on the economy and the level of aggregate employment. We shall examine the use of three types of subsidy – an average cost, a lump-sum and a per-unit subsidy – on the level of economic activity and welfare. Similar to Heijdra *et al* (1998), we assume that in all cases the government uses a proportional tax rate, τ, on consumers' income to finance the subsidy. The rationale for the use of such a tax is two-fold. Firstly, Heijdra *et al* (1998) find that under an income tax, government expenditure actually reduces aggregate output (and thence welfare) due to a crowding out of private consumption; it is therefore of interest to see if this outcome also arises when expenditure takes the form of subsidy provisions instead. Secondly, the use of such a tax makes our analysis intuitively and computationally simple and straightforward[16].

Following the notational convention adopted in Chapter 2, we denote each post-tax variable with a subscript τ, and similarly, post-subsidy variables are denoted with a subscript s to distinguish them from the original set-up. We first redefine the post-tax budget constraint which is now

$$(1-\tau)wL_\tau^S = P_A A + P_Y Y, \tag{3.20}$$

where L_τ^S denotes the post-tax labour supply function.

Maximising utility subject to the new budget constraint in Eq. (3.20) and recalling that $P_A = 1$ and $w = 1$, the labour supply and the demand for goods functions become:

[16] While there are different ways of raising tax revenue, income taxes are one of the principal sources of government revenue in industrial economies. This gives us another rationale to use it.

$$L_\tau^S = \left(\frac{1-\tau}{P_Y^\mu} \right)^{\frac{1}{\gamma}}, \tag{3.21}$$

$$Y_\tau^D = \frac{(1-\tau)\mu L_\tau^S}{P_Y} \tag{3.22}$$

and

$$A_\tau^D = (1-\tau)(1-\mu)L_\tau^S. \tag{3.23}$$

The demand for each individual variety will now be:

$$y_\tau^D = \frac{(1-\tau)\mu L_\tau^S}{P_Y} \left(\frac{p}{P_Y} \right)^{-\sigma}. \tag{3.24}$$

The indirect utility function is now given by:

$$V_\tau = \frac{\gamma}{1+\gamma} \left(\frac{1-\tau}{P_Y^\mu} \right)^{\frac{1+\gamma}{\gamma}}. \tag{3.25}$$

The expressions for the demand side of the model are not affected by the subsidy type as we are assuming the same form of taxation throughout. However, different modes of subsidisation will have a different impact on prices, quantities and firm numbers.

On the production side, we assume that the subsidy is only given to firms in the differentiated sector and none is received by the homogeneous sector. Thus, under perfectly competitive conditions and with a unit labour requirement of one, the market clearing equilibrium for the homogeneous sector is obtained by using Eq. (3.23) with Eq. (3.17) to yield:

$$(1-\tau)(1-\mu)L_\tau^S = A_s^S. \tag{3.26}$$

3.5.1. Average-Cost Subsidies

We now examine each individual subsidy case, beginning with the average-cost subsidy. The average-cost subsidy is essentially an employment subsidy such as that mooted in the literature on regional labour market policies[17], where a firm receives a payment from the government for every unit of labour hired. With the subsidy, each firm's cost and profit functions are now:

$$C_s = \left(\alpha + \beta y_s^S\right)(1-s),\ s<1, \text{ and} \tag{3.27}$$

$$\pi_s = p y_\tau^D - \left(\alpha + \beta y_s^S\right)(1-s), \tag{3.28}$$

where $s<1$ is the subsidy rate. From the cost function in Eq. (3.27), it is straightforward to find that the marginal cost of production is now

$$MC_s = \beta(1-s), \tag{3.29}$$

with the price each firm charges for its output becoming

$$p_s = \frac{\sigma\beta(1-s)}{\sigma-1}. \tag{3.30}$$

Imposing the zero-profit condition, it is easy to verify that each firm's output scale is unaltered, and remains as in Eq. (3.13). As output per firm remains unchanged as per the initial unsubsidised equilibrium, the effect of an average cost subsidy transmits through a reduction in the price of the good as seen from Eq. (3.30).

Combining Eq. (3.13) with the demand for each variety from Eq. (3.24) we obtain the market clearing condition for each variety:

$$\frac{(1-\tau)\mu L_\tau^S}{P_Y}\left(\frac{p}{P_Y}\right)^{-\sigma} = \frac{\alpha(\sigma-1)}{\beta}. \tag{3.31}$$

As there is no subsidy on the homogeneous good's production, the A-good market clearing condition remains as in Eq. (3.26). The labour market equilibrium is now given by:

$$(1-\tau)(1-\mu)L_\tau^S + N_s\alpha\sigma = L_\tau^s. \tag{3.32}$$

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[17] See Kaldor (1970) for example, for a motivation for the use of such region-specific type of policies. For a more empirically based review of labour market policies in practice, see Fischer and Nijkamp (1988) who compare and evaluate the regional labour market policies in a subset of countries or the recommendations by Begg and Portes (1993) with respect to East Germany.

The first term reflects the labour demand in the agricultural sector, while the second is the total labour requirement by firms in the differentiated goods sector. Finally, since firms receive a subsidy for each unit of labour they employ, the government budget constraint is:

$$\tau L_\tau^S = N_s \alpha \sigma s\,, \tag{3.33}$$

where the left-hand side is the government's total tax revenue, while right-hand side is the total subsidy bill[18].

We assume that the government chooses s, the level of subsidy, and lets the tax rate adjust to balance its budget. Using the budget constraint, we obtain the goods and labour market conditions in terms of exogenous variables only, and solve for values of τ and N. Specifically, upon substitution of the expressions for L_τ^S, p and P_Y, the labour market equilibrium, Eq. (3.32) and the government budget constraint, Eq. (3.33), can be re-written as

$$(\mu + (1-\mu)\tau)\left((1-\tau)\left(\frac{(\sigma-1)}{N_s^{\frac{1}{1-\sigma}}\sigma\beta(1-s)}\right)^{\mu}\right)^{\frac{1}{\gamma}} = N_s \alpha \sigma$$

and

$$\tau\left((1-\tau)\left(\frac{(\sigma-1)}{N_s^{\frac{1}{1-\sigma}}\sigma\beta(1-s)}\right)^{\mu}\right)^{\frac{1}{\gamma}} = N_s \alpha \sigma s$$

respectively. We first examine the existence of a stable equilibrium point, and examine qualitatively the effect of a change in subsidy on the equilibrium using the two equilibrium conditions stated. Approximating the two equations to straight lines and plotting them in the N and τ space, we get Figure 3-1.

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[18] The government's budget constraint requires that total revenue meets total expenditure, or $\tau L_\tau^S = N_s s l_Y$. Substituting for $l_Y = \alpha + \beta y$ and $y = \frac{\alpha(\sigma-1)}{\beta}$ and simplifying will give Eq. (3.33).

The vertical arrows show the movement paths above and below the labour market equilibrium locus which we label *LME*. An increase in the tax rate leads to a new point above the locus of the labour market equilibrium. This increase in τ reduces the supply of labour[19] such that firms who are unable to hire enough labour to produce at their optimal scale will incur losses and exit the *Y*-sector. Hence, *N* will fall until the labour market equilibrium is once again restored. The negative slope of the locus is due to the fact that an increase in τ reduces disposable income and the supply of labour. The horizontal arrows show the movements above and below the government budget constraint which we label *GBC*. As a result of an exogenous change in the number of firms *N*, the subsidy bill of the government will change such that it will have to adjust the tax rate to balance in budget. An increase in *N* increases will raise the government's subsidy bill, such that it will need to raise τ to balance its budget, resulting in the *GBC* sloping upwards. The horizontal arrow paths indicate the movement paths above and below the *GBC*.

The intersection, *E*, of the two lines corresponds to the equilibrium values of N_s and τ for a particular *ad hoc* value of *s* where both the *GBC* and *LME* are in equilibrium. From the movement paths of the arrow configurations, we can see that there is a stable and unique equilibrium point. Assuming an increase in *s* as in the dotted lines in Figure 3-1, the *GBC* will shift right to *GBC′* as the government now faces an increased subsidy bill. It must therefore increase the tax rate in order to balance its budget. The *LME* shifts up in response to an increase in labour demand from the now larger number of firms to *LME′*. While the two loci represent all the points where the *GBC* and *LME* are in equilibrium, the resulting position given by *E′* denotes where the conditions for both are met simultaneously.

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19 $\frac{\partial L_\tau^S}{\partial \tau} < 0$ or L_τ^S is decreasing in τ.

Analytically, taking s as exogenous, we can solve for the *ad hoc* equilibrium solutions for N_s and τ to be:

$$N_s = \left(\left(\frac{\sigma - 1}{\sigma\beta} \right)^{\mu} \left(\frac{\mu}{\alpha\sigma} \right)^{\gamma} (1-s)^{1-\mu} (1 - s(1-\mu))^{-(1+\gamma)} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} \tag{3.34}$$

and

$$\tau = \frac{\mu s}{1 - s + \mu s}. \tag{3.35}$$

Given that $\sigma > 1$, we assume the value of γ is sufficiently large to give the condition that $\gamma(\sigma - 1) - \mu > 0$ [20]. Examination of Eq. (3.34) shows that N_s increases with s and decreases with the elasticity of labour supply, $\frac{1}{\gamma}$. A point of further interest to note here is that γ does not appear at all in the tax rate in Eq. (3.35), i.e. the level of tax is independent of the elasticity of labour, with τ being convex with respect to s – as can be seen from Figure 3-2. Intuition may suggest that the labour elasticity, γ, ought to feature in Eq. (3.35). However, as can be seen from the government budget constraint, Eq. (3.33), in this case τ depends on the ratio of N to L which, given the labour market equilibrium, is independent of γ. This result follows since the government budget constraint imposes the homogeneity condition in this case.

The shape of τ implies that the tax rate increases at an increasing rate rather than proportionately with an increase in subsidy levels. More importantly, it suggests that there is a possible welfare maximising level of tax which the government can levy before the benefit of the subsidy is outweighed by the tax burden. The intuition behind this simply lies in the interaction of the tax with the consumer's disposable income and thence, the demand for each variety. As the tax increases, the level of

[20] It is not impossible for $\gamma(\sigma - 1) - \mu < 0$ to hold, but this would imply an almost perfectly elastic labour supply (given that $\sigma > 1$ and $0 < \mu < 1$). We rule this case out by assumption.

disposal income to the consumer is reduced, and this in turn reduces aggregate expenditure across all varieties in the differentiated goods sector. Aggregate demand is, in turn, implicitly tied with the number of firms that can exist in equilibrium as this determines a firm's entry-exit decision[21]. Hence, while a positive tax increases welfare, beyond a certain level, an increase in taxes will reduce welfare instead.

Thus, there may very well be a tax-and-subsidy combination which is welfare maximising[22]. We will return to this later. We first proceed to examine the other effects the tax and subsidy combination has on the economy.

3.5.1.1. Implications

The *ad hoc* equilibrium (i.e. not optimal) values for N_s and τ discussed above are obtained for any given level of s. We can now further examine the implications of these for other variables in the general equilibrium of the model. Firstly, by substituting the equilibrium solutions for N_s and τ into the post-tax labour supply function in Eq. (3.21) and differentiating with respect to s, we get:

$$\frac{\partial L_\tau^S}{\partial s} = \frac{\mu(1-\mu)(1-s\sigma)}{(\gamma(\sigma-1)-\mu)}\left(\frac{\sigma\beta}{\sigma-1}\right)^{\frac{-\mu}{\gamma}}(1-s)^{\frac{1-\mu+\gamma}{\gamma}}(1-s(1-\mu))^{\frac{-(1+\gamma)}{\gamma}}$$

$$\left(\left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu}\left(\frac{\mu}{\alpha\sigma}\right)^{\gamma}(1-s)^{1-\mu}(1-s(1-\mu))^{-(1+\gamma)}\right)^{\frac{\mu}{\gamma(\gamma(\sigma-1)-\mu)}} > 0. \qquad (3.36)$$

Eq. (3.36) implies that total labour supply, and therefore the aggregate level of economic activity, increases as a result of an increase in the subsidy rate. The intuition behind this is simple. From Eq. (3.20), the level of income depends positively with the supply of labour and negatively with the tax rate. Starting from a

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[21] Given that we find the equilibrium output of each firm to remain at pre-subsidy levels after the subsidy provision, the market clearing aggregate supply in the differentiated goods sector will be reached via an adjustment of the number of firms that exist in equilibrium.

[22] This is akin to a Laffer curve type of effect, where an increase in the tax rate beyond a certain point results in utility falling.

zero-subsidy, a rise in s will similarly mean an increase in τ which lowers income. This in turn, induces an individual to increase his labour supply so as to maximise utility

A second implication of the analysis that is worth pointing out is that the subsidy also increases the total number of varieties in general equilibrium. Differentiating N_s with respect to s gives:

$$\frac{\partial N_s}{\partial s} = \frac{(1-\mu)(\sigma-1)(\mu s+\gamma(s-1))}{(s-1)(1-s+s\mu)(\gamma\sigma-\gamma-\mu)}$$

$$\left(\left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu}\left(\frac{\mu}{\alpha\sigma}\right)^{\gamma}(1-s)^{1-\mu}(1-s(1-\mu))^{-(1+\gamma)}\right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} > 0 \qquad (3.37)$$

The larger number of firms also raises aggregate output in the differentiated sector. This higher level of aggregate output together with the larger number of varieties of the differentiated good available for consumption suggests that welfare is raised. This resulting increase in N_s is also influenced by an increase in the labour supply. Recall that in both the unsubsidised, Eq. (3.18), and subsidised, Eq. (3.34), expressions for N, the equilibrium number of varieties is dependent upon the labour supply. As total labour supply has now increased as can be seen from Eq. (3.36), this new increased level of labour supply also supports the larger number of firms now present in the differentiated sector.

There is however, a caveat which needs to be highlighted. It is necessary to remember here that utility is derived from consumption of *both* goods as well as the supply of labour. Thus, while the increase in the total number of varieties suggests a possible welfare improvement, the increase in labour supply works instead against any welfare improvement. Furthermore, labour employment is divided between firms in the Y- and A-sectors. If labour supply is constant, i.e. perfectly inelastic, an increase in labour demand in the *Y*-sector will result in a corresponding decrease in the labour employed and in the output produced in the other sector. This will result in a further lowering of utility due to the lower consumption of the homogenous good. In this case, labour supply is not perfectly inelastic: thus, while total labour supply has increased as $\frac{\partial L_\tau^S}{\partial s} > 0$ and there are a larger number of firms in equilibrium ($\frac{\partial N_s}{\partial s} > 0$)[23] in the differentiated sector, it is as yet unclear how the *A*-sector's output is affected.

Therefore, we also have to examine the impact the subsidy plays on the outcome of labour allocation in the homogeneous sector. Substituting the expressions for N_s and τ into the labour market equilibrium condition and simplifying gives the labour demand in the *A*-sector, L_A. Differentiating with respect to *s*, we obtain:

$$\frac{\partial L_A}{\partial s} = \frac{\mu(1-\mu)(1-s\sigma)}{(1-s(1-\mu))(1-s)(\gamma(\sigma-1)-\mu)}\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{\frac{-\mu}{\gamma}}$$

$$\left(\left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu}\left(\frac{\mu}{\alpha\sigma}\right)^{\gamma}(1-s)^{1-\mu}(1-s(1-\mu))^{-(1+\gamma)}\right)^{\frac{\mu}{\gamma(\gamma(\sigma-1)-\mu)}}$$

$$+\frac{\alpha\sigma(\mu-1)(\mu s+\gamma(s-1))}{(1-s(1-\mu))(s-1)(\gamma(\sigma-1)-\mu)}$$

$$\left(\left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu}\left(\frac{\mu}{\alpha\sigma}\right)^{\gamma}(1-s)^{1-\mu}(1-s(1-\mu))^{-(1+\gamma)}\right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} < 0\,, \tag{3.38}$$

where L_A refers to the labour demand in the *A*-sector. Thus, while total labour supply is now higher, its total increase is smaller than the increment in labour demand from the *Y*-sector. As a result, as in the case with inelastic labour supply, some labour reallocation between sectors has actually taken place. The reduction in the labour

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[23] As $L_Y = N_s l_Y$, an increase in N_s will mean a corresponding increase in L_Y since l_Y is a constant at $l_Y = \alpha + \beta y$.

allocation to the A-sector also implies that output, and consumption, of the homogenous good is now lower. This will offset to some extent the increase in utility stemming from the larger number of varieties of the differentiated goods.

Thus, we have two effects working against each other, reduced consumption of the A-good which lowers utility, and an increase in the number of varieties available to the consumer which raises utility. Given too that the supply of labour, which reduces utility, has increased as well, the net outcome on welfare is, qualitatively speaking, somewhat ambiguous. To determine the actual effect of the tax-subsidy combination, we return to the indirect utility function. Using the expressions for N_s and τ, we substitute them into V_τ and examine the effect of a small change in subsidy has on welfare, i.e. $s = 0$. Differentiating with respect to s and evaluating at $s = 0$, this yields:

$$\left.\frac{\partial V_\tau}{\partial s}\right|_{s=0} = \frac{\mu\gamma(1-\mu)}{\gamma(\sigma-1)-\mu}\left(\left(\frac{\sigma\beta}{\sigma-1}\right)^{\sigma-1}\left(\frac{\alpha\sigma}{\mu}\right)\right)^{\frac{-\mu(1+\gamma)}{\gamma(\sigma-1)-\mu}} > 0. \tag{3.39}$$

This therefore suggests that the implementation of an average-cost subsidy is potentially welfare improving. Also, it implies that $s = 0$ is not a socially optimal policy rule and that there exists a socially-optimal tax-subsidy combination that the government can implement.

3.5.1.2. The Optimal Subsidy

The optimal policy rule can be obtained by setting $\frac{\partial V_\tau}{\partial s}$ to zero, and solving for the optimal subsidy, s^*, which maximises utility. The value of the optimal subsidy is found to be

$$s^* = \frac{1}{\sigma}, \tag{3.40}$$

which is clearly positive. To ensure that the optimal value of the subsidy obtained is a maximum rather than a minimum, differentiating the indirect utility function twice and substituting s^* into the resulting expression yields:

$$\left.\frac{\partial^2 V_\tau}{\partial s^2}\right|_{s=s^*} = \frac{\mu\gamma\sigma^3(\mu-1)}{(\sigma-1+\mu)^2(\gamma(\sigma-1)-\mu)}\left(\frac{\sigma-1}{\sigma-1+\mu}\right)^{\frac{1}{\gamma}}$$

$$\left(\beta\left(\left(\frac{\mu}{\alpha}\right)^{\gamma}\beta^{-\mu}(\sigma-1)(\sigma-1+\mu)^{-(1+\gamma)}\right)^{\frac{-1}{\gamma(\sigma-1)-\mu}}\right)^{\frac{-\mu(1+\gamma)}{\gamma}} < 0, \tag{3.41}$$

which satisfies the second order condition for welfare maximisation.

Next, we also obtain the number of firms and tax rate that correspond to the optimal subsidy by substituting s^* into the expression for N_s in Eq. (3.34), and τ into Eq. (3.35). This gives:

$$N_s^* = \left(\left(\frac{\mu}{\alpha}\right)^{\gamma}(\sigma-1)\beta^{-\mu}(\sigma-1+\mu)^{-(1+\gamma)}\right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}}, \tag{3.42}$$

$$\tau^* = \frac{\mu}{\sigma-1+\mu}. \tag{3.43}$$

We find also the optimal value of the subsidy to be independent of all other factors and parameters other than the elasticity of substitution between varieties. Thus, the value of s^* falls as each variety becomes a closer substitute to the next, and vice versa. In the polar cases where $\sigma \to \infty$ or $\sigma \to 1$, the mark-up over marginal cost becomes $\frac{\sigma}{\sigma-1} \to 1$ and $\frac{\sigma}{\sigma-1} \to \infty$, with $s^* \to 0$ and $s^* \to 1$ respectively.

The rationale behind this is straightforward. As σ increases, varieties become closer substitutes to each other and the utility gain that consumers get from the introduction of extra varieties falls. Hence, the degree of sub-optimality in the production of varieties falls – but so does the optimal value of the optimal subsidy (whose aim is to correct the underproduction of varieties).

Summing up, an average cost subsidy raises the total level of economic activity in the economy as total labour supply increases. It also increases the number of varieties of differentiated goods available for consumption while reducing the price of each variety in the differentiated good sector. There is however, reallocation of labour from the homogeneous to the differentiated goods sector, giving rise to a reduction in the output (and consumption) in the homogeneous sector. In spite of this fall however, its use is found to be overall welfare improving.

3.5.2. Lump-Sum Handouts

We now examine the effect of the use of a lump-sum subsidy, which entails the following modification to the cost and profit functions of the typical firm:

$$C_s = \left(\alpha + \beta y_s^S\right) - s \tag{3.44}$$

and

$$\pi_s = p_s y_s^D - \left(\alpha + \beta y_s^S\right) + s\,. \tag{3.45}$$

The solution to the firm's maximisation problem yields the optimal price rule:

$$p_s = \frac{\sigma\beta}{\sigma - 1} \tag{3.46}$$

Given, Eq. (3.46), imposition of the zero-profit condition gives the following optimal output scale:

$$y_s^S = \frac{(\alpha - s)(\sigma - 1)}{\beta} \tag{3.47}$$

Using these together with the post-tax demand equations, we obtain the equilibrium conditions as follow. The differentiated goods market clearing now requires:

$$\frac{(1-\tau)\mu L_\tau^S}{P_Y}\left(\frac{p_s}{P_Y}\right)^{-\sigma} = \frac{(\alpha - s)(\sigma - 1)}{\beta}; \tag{3.48}$$

the homogeneous sector equilibrium condition is as in Eq. (3.26):

$$(1-\tau)(1-\mu)L_\tau^S = A_s^S\,.$$

The labour market equilibrium condition is given by

$$(1-\tau)(1-\mu)L_{\tau}^{S}+N_{s}\left(\alpha\sigma-s(\sigma-1)\right)=L_{\tau}^{S}\,. \tag{3.49}$$

Finally, assuming that the government practises a balanced budget rule, the government's budget constraint is defined as

$$\tau L_{\tau}^{S}=N_{s}s \tag{3.50}$$

where total tax revenue from labour income equals the total value of the subsidy bill paid out to firms. We assume the government fixes the subsidy and lets the tax rate vary to balance the budget, with N and τ determined endogenously from the equilibrium conditions. Using the relevant expressions for L_{τ}^{S}, p and P_Y, we can rewrite the labour market equilibrium condition and the government budget constraint as:

$$N_{s}\left(\alpha\sigma-s(\sigma-1)\right)=\left((1-\tau)\left(\frac{(\sigma-1)}{N_{s}^{\frac{1}{1-\sigma}}\sigma\beta}\right)^{\mu}\right)^{\frac{1}{\gamma}}\left(\tau(1-\mu)+\mu\right)\text{ and}$$

$$\tau\left((1-\tau)\left(\frac{(\sigma-1)}{N_{s}^{\frac{1}{1-\sigma}}\sigma\beta}\right)^{\mu}\right)^{\frac{1}{\gamma}}=N_{s}s\,.$$

These can be used to determine the *ad hoc* (i.e. non-optimal) equilibrium values of N_s and τ. As before, we first check for the existence of a stable equilibrium. The labour market equilibrium and the government budget constraint are both sketched in Figure 3-3, where the locus of each shows all combinations of N and τ where each individual condition is satisfied. The intuition behind the slopes of each is qualitatively similar to that for the average cost subsidy case. The intersection, E, of the loci corresponds to a general equilibrium point. As before, the effect of a change in the subsidy level can be examined qualitatively. Starting with a given increase in s, the *LME* and *GBC* will shift rightwards to *LME′* and *GBC′* respectively. The directions of the arrows denote the movement paths above and below each line, and we see the result of an increase in *N* subsequently results in the convergence to a new equilibrium point given by *E′*.

Solving for the general equilibrium solutions for N_s and τ, these are found to be

$$N_s = \left(\mu^{\gamma} (\alpha - s)\sigma \left(\frac{(\sigma - 1)}{\sigma\beta} \right)^{\mu} (\alpha\sigma - s(\sigma - \mu))^{-(1+\gamma)} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} \tag{3.51}$$

$$\tau = \frac{\mu s}{\alpha\sigma - s(\sigma - \mu)} \tag{3.52}$$

The number of firms in equilibrium is increasing in s and γ. The tax rate is, as in the case of the average cost subsidy, strictly positive and independent of γ. The value of s in this case lies in the range $(0, \alpha)$: thus substituting for $s = 0.25\alpha$, $s = 0.5\alpha$ and $s = \alpha$, the corresponding values of τ are $\tau_{s=0.25\alpha} = \frac{\mu}{3\sigma+\mu}$, $\tau_{s=0.5\alpha} = \frac{\mu}{\sigma+\mu}$ and $\tau_{s=\alpha} = 1$. Using these values, the corresponding shape of the tax-subsidy schedule is similar to that illustrated in Figure 3-2.

3.5.2.1. Related Findings

From Figure 3-3, we see that the number of firms in the differentiated sector increases as a result of the subsidy. This is confirmed when we differentiate N_s with respect to s:

$$\frac{\partial N_s}{\partial s} = \frac{(\sigma - 1)(\gamma(\alpha - s)(\sigma - \mu) - \alpha\mu)}{(\alpha - s)(\gamma(\sigma - 1) - \mu)(\alpha\sigma - s(\sigma - \mu))}$$

$$\left(\mu^{\gamma} (\alpha - s)\sigma \left(\frac{\sigma - 1}{\sigma\beta} \right)^{\mu} (\alpha\sigma - s(\sigma - \mu))^{-(1+\gamma)} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} > 0. \tag{3.53}$$

In the case of the average-cost subsidy, the increase in the number of firms corresponded to a higher total labour supply. We examine if the same happens here.

Substituting N_s, P_Y and p into L_τ^S and differentiating with respect to s, we get:

$$\frac{\partial L_\tau^S}{\partial s} = \frac{\mu(\alpha(1-\mu) - s(\sigma-\mu))}{(\alpha - s)(\alpha\sigma - s(\sigma-\mu))(\gamma(\sigma-1)-\mu)}$$

$$\left((\alpha\sigma - s(\sigma-\mu))^{1-\sigma-\mu}((\alpha - s)\sigma)^{\sigma-1}\mu^{\mu}\left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu(\sigma-1)}\right)^{\frac{1}{\gamma(\sigma-1)-\mu}} > 0 \qquad (3.54)$$

which implies that the total supply of labour is also higher than the pre-subsidy level. Note however, that for this to be the case, the condition $\frac{\alpha}{s} \geq \frac{\sigma-\mu}{1-\mu}$ needs to hold. If the ratio of fixed costs to the lump-sum subsidy is $\frac{\alpha}{s} < \frac{\sigma-\mu}{1-\mu}$ instead, labour supply will fall as a result. Thus, there is some critical value of α up to which the government can increase the subsidy without incurring in a reduction in labour supply.

As discussed for the case of average cost subsidy, it is not easy to determine *a priori* whether an increase in labour supply results in higher output levels in the homogeneous good sector. Substituting for the values of N_s and τ into L_A, the change in labour supply in the A-sector as a result of the subsidy is found to be

$$\frac{\partial L_A}{\partial s} = \frac{\mu\sigma(\mu-1)(\alpha(\gamma(\sigma-1)-1) + s(\sigma-\mu))}{(\alpha\sigma - s(\sigma-\mu))^2(\gamma(\sigma-1)-\mu)}$$

$$\left((\alpha\sigma - s(\sigma-\mu))^{1-\sigma-\mu}((\alpha - s)\sigma)^{\sigma-1}\mu^{\mu}\left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu(\sigma-1)}\right)^{\frac{1}{\gamma(\sigma-1)-\mu}} < 0. \qquad (3.55)$$

Given Eq. (3.17), it is easy to verify that the total output of A has decreased as a result. Therefore, a lump sum subsidy also brings about a similar outcome as the average cost subsidy by lowering output of the homogeneous sector. This reduction in labour in the A-sector also implies that some labour reallocation has taken place. Intuitively, the expansion in the number of firms in the differentiated sector leads to a correspondingly higher labour demand. This increase overrides the total increase in labour supply, such that some labour reallocation from the *A*- to the *Y*-sector takes place as a result.

The total labour demand and the number of firms in the *Y*-sector are now higher; however, we need to determine how the net aggregate output of the sector changes in response to the subsidy provision. In the average cost case, output in the *Y*-sector unambiguously expanded, given that the subsidy led to an increase in the number of firms without affecting (in comparison to the no-subsidy case) the optimal output scale at the firm level. In this case, however, we have an increase in the number of firms, but each firm now has a smaller equilibrium output scale relative to the unsubsidised equilibrium. Aggregate output in the sector is found as $Y_s = N_s y_s^S$ or, substituting for N_s and y_s^S,

$$Y_s = \left(\mu^\gamma (\alpha - s)\sigma \left(\frac{(\sigma-1)}{\sigma\beta} \right)^\mu (\alpha\sigma - s(\sigma-\mu))^{-(1+\gamma)} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} \frac{(\alpha-s)(\sigma-1)}{\beta}. \tag{3.56}$$

Differentiating with respect to s, we get

$$\frac{\partial Y_s}{\partial s} = \frac{\mu(1-\sigma)(\alpha(\gamma(\sigma-1)-1)+s(\sigma-\mu))}{\beta(\gamma(\sigma-1)-\mu)(\alpha\sigma - s(\sigma-\mu))}$$

$$\left(\mu^\gamma (\alpha - s)\sigma \left(\frac{\sigma-1}{\sigma\beta} \right)^\mu (\alpha\sigma - s(\sigma-\mu))^{-(1+\gamma)} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}}. \tag{3.57}$$

Recall that we had previously made the assumption that $\gamma(\sigma-1)-\mu > 0$: as a result, $\gamma(\sigma-1)-1 > 0$ [24] which implies that $\frac{\partial Y_s}{\partial s} < 0$. Therefore, unlike the case of the average-cost subsidy, another consequence of the lump-sum subsidy is the contraction in the level of output in the differentiated sector. This fall coupled with the reduced *A*-good consumption, renders the welfare effects of the subsidy

[24] Recall that μ lies between 0 and 1. Thus, if $\gamma(\sigma-1)-\mu > 0$, in the polar case of $\mu = 1$, $\gamma(\sigma-1)-1$ is also greater than 0.

somewhat ambiguous[25].

As with the average-cost subsidy, the shape of the tax-subsidy schedule from the solution of τ suggests the possible existence of an optimal tax-subsidy combination. We first examine the effect of a small change in subsidy. Substituting the values of N_s and τ into the indirect utility function and taking the first derivative with respect to s and evaluating at $s = 0$, we obtain:

$$\left.\frac{\partial V_\tau}{\partial s}\right|_{s=0} = \frac{\mu\gamma(1-\mu)}{\alpha\sigma(\gamma(\sigma-1)-\mu)}\left(\left(\frac{\sigma\beta}{\sigma-1}\right)^{\sigma-1}\left(\frac{\alpha\sigma}{\mu}\right)\right)^{\frac{-\mu(1+\gamma)}{\gamma(\sigma-1)-\mu}} > 0. \tag{3.58}$$

Eq. (3.58) suggests that: (i) $s = 0$ is not a welfare maximising policy rule as a small deviation from it is welfare improving; (ii) the effect of increase in the number of varieties on welfare more than compensates the welfare effect of the fall in the level of aggregate output (and consumption) of both the *Y*- and *A*-goods.

3.5.2.2. The Optimal Subsidy

As with the average cost subsidy, we determine the value of the optimal subsidy by setting $\frac{\partial V_\tau}{\partial s} = 0$ and solving for s^* to obtain:

$$s^* = \frac{\alpha(1-\mu)}{\sigma-\mu}. \tag{3.59}$$

From Eq. (3.59), the size of the optimal subsidy is dependent on the level of fixed costs, consumption share, as well as the elasticity of substitution between varieties. Furthermore, the portion of fixed costs to be subsidised is a fraction, $\frac{1-\mu}{\sigma-\mu}$, of α, which clearly, is always positive since $\mu < 1$.

An observation that is of particular interest here follows from the change in

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[25] A larger number of varieties of the *Y*-good are available for consumption and this suggests a welfare improvement. However, there is a simultaneous reduction in the quantity consumed of both the differentiated as well as the homogeneous good. The reduction in consumption quantity clearly works towards an offset of any welfare gain resulting from the variety effect of the policy.

labour supply found from Eq. (3.54). We noted that there is a range in the level of the lump-sum subsidy which will raise the labour supply before labour supply starts to fall. In this case, the value of s^* indicates that the optimal subsidy not only maximises welfare, it also maximises the labour supply. Substituting s^* into Eq. (3.54) gives the change in labour supply as $\frac{\partial L_\tau^S}{\partial s} = 0$. Thus, this implies that the optimal subsidy also gives the maximum quantity of labour supply in the economy.

We can now obtain the tax rate that corresponds to the optimal subsidy, τ^*, by substituting the expression for s^* into τ:

$$\tau^* = \frac{\mu(1-\mu)}{(\sigma-\mu)(\sigma-1+\mu)}. \tag{3.60}$$

Similarly, the corresponding number of firms in the post-subsidy equilibrium is found to be

$$N_s^* = \left(\left(\frac{\mu}{\alpha}\right)^{\gamma} \frac{\sigma(\sigma-1)}{(\sigma-\mu)} \left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu} (\sigma-1+\mu)^{-(1+\gamma)} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}}. \tag{3.61}$$

Finally, differentiating $\frac{\partial V_\tau}{\partial s}$ again by s and evaluating at s^* gives

$$\left.\frac{\partial^2 V_\tau}{\partial s^2}\right|_{s=s^*} = \frac{\mu\gamma\sigma(\mu-\sigma)}{\alpha^2(\sigma-1+\mu)^2(\gamma(\sigma-1)-\mu)} \left(\frac{\sigma(\sigma-1)}{(\sigma-1+\mu)(\sigma-\mu)}\right)^{\frac{1}{\gamma}} \left(\frac{\sigma-1}{\sigma\beta}\right)^{\frac{\mu(1+\gamma)}{\gamma}}$$

$$\left(\left(\frac{\mu}{\alpha}\right)^{\gamma} \frac{\sigma(\sigma-1)}{(\sigma-\mu)} (\sigma-1+\mu)^{-(1+\gamma)} \left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu} \right)^{\frac{\mu(1+\gamma)}{\gamma(\gamma(\sigma-1)-\mu)}} < 0. \tag{3.62}$$

This ensures the policy rule obtained corresponds to a maximum point, and welfare does indeed rise to a higher level with the subsidy.

3.5.3. Per-Unit (Production) Payments

We now look at the effect of a subsidy given for each unit of output. The total cost and profit function corresponding to this care are respectively given by:

$$C_s = \left(\alpha + \beta y_s^S\right) - sy_s \tag{3.63}$$

$$\pi_s = p_s y_\tau^D - \left(\alpha + \beta y_s^S\right) + sy_s^S . \tag{3.64}$$

Similar to the average cost subsidy case, a per-unit subsidy changes the marginal cost of production, and therefore the optimal price rule of the firm as well. These are respectively given by:

$$MC_s = \beta - s . \tag{3.65}$$

and

$$p_s = \frac{\sigma(\beta - s)}{\sigma - 1}. \tag{3.66}$$

The equilibrium level of output per firm, upon imposing the zero-profit condition, increases in this case to

$$y_s^S = \frac{\alpha(\sigma - 1)}{\beta - s}. \tag{3.67}$$

Unlike the average cost and lump-sum subsidies where the effect of the subsidy to the firm is reflected primarily in either the equilibrium price charged or the quantity of output produced, the production subsidy works via both the equilibrium price, as seen in Eq. (3.66) and the optimum output level, Eq. (3.67).

The market clearing conditions for the differentiated good requires $y_s^S = y_\tau^D$, i.e.:

$$\frac{(1-\tau)\mu L_\tau^S}{P_Y}\left(\frac{p}{P_Y}\right)^{-\sigma} = \frac{\alpha(\sigma - 1)}{\beta - s}, \tag{3.68}$$

while the homogeneous goods market equilibrium follows from Eq. (3.26) again, or

$$(1-\tau)(1-\mu)L_\tau^S = A_s^S .$$

The labour market equilibrium requires:

$$(1-\tau)(1-\mu)L_\tau^S + N_s \frac{\alpha(\sigma\beta - s)}{\beta - s} = L_\tau^S \tag{3.69}$$

in this case. As the subsidy is paid towards every unit of output produced by the firms, the government's budget constraint in this case is given by:

$$\tau L_\tau^S = N_s s y_s$$

$$= N_s s \frac{\alpha(\sigma-1)}{\beta-s}. \tag{3.70}$$

Using the differentiated goods market equilibrium condition and the government budget constraint, the qualitative analysis for the existence of a stable equilibrium is shown in Figure 3-4. As in the previous cases, the arrows indicate the movement paths above and below each equilibrium condition. Note that unlike the two previous cases, when τ increases following a raise in s, the figure suggests that the number of firms in equilibrium could *fall*.

The underlying rationale for the positive slope of the *GBC* is identical to the previous cases. The underlying intuition for the slope of the differentiated goods market equilibrium locus which we label as *DGME* is similar to the *LME* in the previous two cases. A change in τ results in a change in the level of disposable income and subsequently affects demand. Assuming a rise in τ, this brings us to a point above the *DGME*. Firms' profits will fall and lead to exit from the industry until the equilibrium in the differentiated goods market is once again restored. The reverse holds for a fall in the tax rate.

Solving for the general equilibrium values of N_s and τ for a given level of subsidy, we obtain:

$$N_s = \left(\left(\frac{\sigma-1}{\sigma(\beta-s)} \right)^{\mu} \left(\frac{\sigma(\beta-s)}{\sigma\beta - s(\mu+\sigma(1-\mu))} \right)^{1+\gamma} \left(\frac{\mu}{\alpha\sigma} \right)^{\gamma} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} \tag{3.71}$$

$$\tau = \frac{\mu s(\sigma-1)}{\sigma\beta - s(\mu+\sigma(1-\mu))} \tag{3.72}$$

Using *ad hoc* values of *s* to obtain the tax rates for $s = 0.25\beta$, $s = 0.5\beta$, and $s = \beta$, the corresponding values of τ are $\tau_{s=0.25\beta} = \frac{\mu(\sigma-1)}{\mu(\sigma-1)+3\sigma}$, $\tau_{s=0.5\beta} = \frac{\mu(\sigma-1)}{\mu(\sigma-1)+\sigma}$ and $\tau_{s=\beta} = 1$ respectively. The resulting tax-subsidy schedule can again be illustrated to have the shape as Figure 3-2.

3.5.3.1. Comparative Statics

Figure 3-4 suggests that the number of firms in the differentiated sector could fall as a result of the subsidy – which, given consumers love-of-variety, clearly works towards a reduction in welfare. To investigate this, we substitute for N_s and τ into V_s and differentiate with respect to *s*. We then evaluate this derivative at $s = 0$ to obtain:

$$\left.\frac{\partial V_\tau}{\partial s}\right|_{s=0} = \frac{\mu\gamma(1-\mu)(\sigma-1)}{\sigma\beta(\gamma(\sigma-1)-\mu)}\left(\left(\frac{\sigma\beta}{\sigma-1}\right)^{\sigma-1}\left(\frac{\alpha\sigma}{\mu}\right)\right)^{\frac{-\mu(1+\gamma)}{\gamma(\sigma-1)-\mu}} > 0. \tag{3.73}$$

Clearly, despite the possible fall in *N*, there is still a welfare gain from using the subsidy. This confirms the results of both Ng and Zhang (2007) and Johdo (2008) that the use of a production subsidy is indeed welfare improving.

We can verify too that similar to Ng and Zhang (2007), we have a fall in the equilibrium number of firms in the differentiated sector[26]. Differentiating Eq. (3.71) with respect to *s*, we get

$$\frac{\partial N_s}{\partial s} = \frac{\mu(1-\sigma)(\beta(\gamma(\sigma-1)-1)+s(\mu+\sigma(1-\mu)))}{(\beta-s)(\gamma(\sigma-1)-\mu)(\sigma\beta-s(\mu+\sigma(1-\mu)))}$$

$$\left(\left(\frac{\mu}{\alpha\sigma}\right)^{\gamma}\left(\frac{\sigma-1}{\sigma(\beta-s)}\right)^{\mu}\left(\frac{\sigma(\beta-s)}{\sigma\beta-s(\mu+\sigma(1-\mu))}\right)^{1+\gamma}\right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} < 0. \tag{3.74}$$

Aggregate output in the differentiated sector is given by

[26] Johdo (2008) reaches a different conclusion. Unlike our case where there is full employment from the outset, his initial assumption is that there is an excess of labour supply within the economy. The subsidy raises individual firms' output and induces the new entry of firms. The combination of both effects subsequently allows full employment to be reached in the labour market.

$$Y_s = \left(\left(\frac{\mu}{\alpha\sigma} \right)^{\gamma} \left(\frac{\sigma - 1}{\sigma(\beta - s)} \right)^{\mu} \left(\frac{\sigma(\beta - s)}{\sigma\beta - s(\mu + \sigma(1-\mu))} \right)^{1+\gamma} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} \left(\frac{\alpha(\sigma - 1)}{\beta - s} \right), \qquad (3.75)$$

with the corresponding change in output found to be

$$\frac{\partial Y_s}{\partial s} = \frac{\alpha(\sigma-1)((\mu+\sigma(1-\mu))(\gamma(\sigma-1)(\beta-s)+\mu s(\sigma+2))-\mu\beta)}{(\gamma(\sigma-1)-\mu)(\beta-s)^2(\sigma\beta-s(\mu+\sigma(1-\mu)))}$$

$$\left(\left(\frac{\mu}{\alpha\sigma} \right)^{\gamma} \left(\frac{\sigma - 1}{\sigma(\beta - s)} \right)^{\mu} \left(\frac{\sigma(\beta - s)}{\sigma\beta - s(\mu + \sigma(1-\mu))} \right)^{1+\gamma} \right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} > 0. \qquad (3.76)$$

Thus, while the number of firms is now lower, the increase in output per firm as a result of the subsidy in the differentiated sector raises overall aggregate output, i.e. there is a larger quantity of each existing variety produced despite the fall in the total number of varieties available.

It is interesting to verify the effects of the subsidy on labour supply by substituting for N_s, τ and p_s into L_τ^S and differentiating with respect to s. This gives

$$\frac{\partial L_\tau^S}{\partial s} = \frac{(\sigma-1)(\beta(1-\mu)-s(\mu+\sigma(1-\mu)))}{(\beta-s)(\gamma(\sigma-1)-\mu)(\sigma\beta-s(\mu+\sigma(1-\mu)))}$$

$$\left(\left(\frac{\sigma(\beta-s)}{\sigma-1} \right)^{\frac{\mu(\gamma(\sigma-1)-2\mu)}{\gamma}} \left(\frac{\sigma(\beta - s)}{\sigma\beta - s(\mu + \sigma(1-\mu))} \right)^{\sigma-1+\mu} \left(\frac{\mu}{\alpha\sigma} \right)^{\mu} \right)^{\frac{1}{\gamma(\sigma-1)-\mu}} > 0 \quad (3.77)$$

Thus, while the individual number of firms demanding labour has decreased, total labour supply as increased[27] as a result of the tax imposition needed to fund the subsidy.

Finally, from the market clearing condition for the *A*-sector in Eq. (3.23), we can analyse the effects of the subsidy on this sector's output:

[27] Note here that the critical value of variable cost to be subsidised is between 0 and $\frac{\mu+\sigma(1-\mu)}{1-\mu}$. Beyond this, labour supply will fall. This is similar to the case of the lump-sum subsidy.

$$\frac{\partial A_s^S}{\partial s}=\frac{\mu\sigma(\mu-1)(\sigma-1)(\beta(\gamma(\sigma-1)-1)+s(\mu+\sigma(1-\mu)))}{(\gamma(\sigma-1)-\mu)(\sigma\beta-s(\mu+\sigma(1-\mu)))^2}$$

$$\left(\left(\frac{\sigma(\beta-s)}{\sigma-1}\right)^{\frac{\mu(\gamma(\sigma-1)-2\mu)}{\gamma}}\left(\frac{\sigma(\beta-s)}{\sigma\beta-s(\mu+\sigma(1-\mu))}\right)^{\sigma-1+\mu}\left(\frac{\mu}{\alpha\sigma}\right)^{\mu}\right)^{\frac{1}{\gamma(\sigma-1)-\mu}}<0 \quad (3.78)$$

Eq. (3.78) indicates that output of the A-good has subsequently fallen. And following on from the production function of the A-good, this implies that some degree of labour reallocation has taken place between sectors.

In conclusion, while there is a reduction of the number of varieties of the Y-good, total output of the A-good and an increase in the labour supply, there is still a positive welfare gain from the implementation of the subsidy as shown by Eq. (3.73). We now turn to the determination of the optimal subsidy rule.

3.5.3.2. The Optimal Subsidy

By setting the expression for $\frac{\partial V_\tau}{\partial s}=0$, the optimal subsidy s is found to be:

$$s^*=\frac{\beta(1-\mu)}{\sigma(1-\mu)+\mu}. \quad (3.79)$$

As in the other two cases, it is evident from the above expression that the value of the subsidy is positive. Also, it is easy to verify that the value of the optimal subsidy corresponds to the level at which labour supply in the economy is at its highest.

Substituting for s^* into τ and N_s, we get the general equilibrium values of the tax rate and the number of firms that correspond to the optimal subsidy policy:

$$\tau^*=\frac{\mu(1-\mu)(\sigma-1)}{(\sigma(1-\mu)+\mu)(\sigma-1+\mu)} \quad (3.80)$$

and

$$N_s^*=\left(\left(\frac{\mu}{\alpha\sigma}\right)^{\gamma}\left(\frac{\sigma(\sigma(1-\mu)-1+2\mu)}{(\sigma-1+\mu)(\mu+\sigma(1-\mu))}\right)^{1+\gamma}\left(\frac{(\sigma-1)(\mu+\sigma(1-\mu))}{\sigma\beta(\sigma(1-\mu)-1+2\mu)}\right)^{\mu}\right)^{\frac{\sigma-1}{\gamma(\sigma-1)-\mu}} \quad (3.81)$$

Finally, to ensure that policy corresponds to a maximum point is the result of the policy, the second derivative of the indirect utility function when evaluated at the optimal subsidy gives

$$\left.\frac{\partial^2 V_\tau}{\partial s^2}\right|_{s=s^*} = \frac{\gamma\mu\sigma(\sigma-1)(\mu(\sigma-1)-\sigma)}{\beta^2(\sigma-1+\mu)^2(\gamma(\sigma-1)-\mu)}\left(\frac{\sigma(\mu(\sigma-2)-\sigma+1)}{(\sigma-1+\mu)(\mu(\sigma-1)-\sigma)}\right)^{\frac{1}{\gamma}}$$

$$\left(\left(\frac{\sigma-1}{\sigma\beta}\right)^{\gamma(\sigma-1)-1}\left(\frac{\mu(\sigma-1)-\sigma}{\mu(\sigma-2)-\sigma+1}\right)^{\gamma(\sigma-2)-1}\left(\frac{\mu}{\alpha}\right)^{\gamma}\beta^{-1}\right)^{\frac{\mu(1+\gamma)}{\gamma(\gamma\sigma-\gamma-\mu)}} < 0\,, \qquad (3.82)$$

confirming that the second-order condition for a maximum is satisfied.

3.6. Subsidy Comparison and Policy Implications

We have so far derived the effects of each class of subsidies in Section 3.5 along with the optimal policy rules. We now turn to the more practical relevance of our results in terms of potential policy implications. Recapitulating, the primary finding is that the use of a tax and subsidy policy raises welfare. This is evident from the fact that for all 3 subsidy types $\frac{\partial V_\tau}{\partial s} > 0$ holds at the $s = 0$ point. Thus, our results support the view that an active industrial policy can be welfare improving.

Denoting the average cost, lump sum and per unit subsidies with the subscripts *AC*, *LS* and *PU* respectively, we briefly consider some possible policy rationales that determine the subsidy choice used. Following Bénassy (1993), suppose that a benevolent government wants to increase labour supply in the belief that it raises welfare. Assume that all three subsidies are feasible options. We know that all three subsidies raise L^S, albeit in varying degrees. However, it easy to show that evaluating the derivative of the welfare function with respect to the subsidy at $s = 0$ yields the following ranking of effects:

$$\frac{\partial V_{LS}}{\partial s} < \frac{\partial V_{PU}}{\partial s} < \frac{\partial V_{AC}}{\partial s}.$$

Thus, the average-cost subsidy would be the preferred policy of a welfare-maximising government. In this set up, this is equivalent to a wage subsidy.

A note-worthy point here concerns the specification of our utility function. One could question how our results may change if a Cobb-Douglas utility function as in Mankiw (1988) and Startz (1989) were used instead. As we had assumed that there is excess (un-utilised) labour, we are always on the upward sloping portion of the labour supply curve. Thus, allowing for an income effect will not change the qualitative nature of our results. However, what may differ is the magnitude of the change in labour supply, i.e. the size of $\frac{\partial L_{\tau}^{S}}{\partial s}$.

Next, if we assume that the government has full information as to which policy creates the greatest welfare gain after implementation, then the preferred subsidy to use would again be the average-cost subsidy. Clearly, there may be issues concerning the realism of such an assumption. As Buchanan (1959) had argued, what constitutes welfare improvements is very often a subjective evaluation depending on each individual's personal criteria. When policy making is subject to the outcomes of majority voting, it may not always be the case that the 'ideal' policy would be implemented.

From our results, we know there is always a Pareto improvement regardless of the subsidy used. However, if the government is not aware of the welfare ordering as stated previously, it may be more inclined to simply measure welfare using more 'visible' indicators such as the number of varieties or aggregate output. Thence, it is clear that a government who is more concerned with raising the number of varieties of the differentiated good will favour the lump-sum and average-cost subsidies over the production subsidy[28]. Or, if increasing consumption quantity of each available variety be the aim, then clearly, the production subsidy is the policy instrument to be used.

As an alternative, we also consider the general equilibrium tax rate (τ^*) that corresponds to the optimal subsidy as a decision criterion for the policy choice. Using this, there is an implicit assumption that the government has complete information and is always able to set its tax-subsidy combination at the welfare maximising level regardless of which subsidy it chooses. Again using the subscripts *AC*, *LS* and *PU* to denote the average cost, lump sum and per-unit subsidies respectively, we can rank the optimal tax rates that correspond to each from the smallest to the largest. This gives:

$$\tau_{LS}^* < \tau_{PU}^* < \tau_{AC}^* .$$

While the tax rate for a lump-sum subsidy is the lowest, so is the corresponding welfare gain.

An interesting point here is how our results relate to those of Fuest and Huber (2000), who find empirical evidence to suggest that most governments appear not to opt for the use of wage subsidies, despite this being the most welfare-enhancing. They postulate this particular preference is due to the presence of union bargaining power in heterogeneous firms which raises the costs of a wage subsidy. Yet, taken on face value, it may simply be due to the lower tax rate which could make such a policy more appealing[29].

---

[28] Recall that $\frac{\partial N_s}{\partial s} > 0$ for both the average cost and the lump sum subsidies, but $\frac{\partial N_s}{\partial s} < 0$ for the per unit case.

[29] The reasoning behind this falls under public choice theory and is out of the purview of this chapter. As an initial exploration however, consider the tax rate as the ‘price’ an electorate has to pay for voting a government to power. Thus, a government seeking to maximise both votes *and* welfare could plausibly opt for the policy with the lowest tax instead, so as to increase its chances of being in power. In our analysis, this falls on the lump sum subsidy and tax combination. For a more complete analysis, see Winer and Hettich (2004).

3.7. Closing Comments

The New Keynesian macroeconomic literature holds that fiscal policy in the form of the government acting as a consumer is a useful policy tool for raising social welfare. In this chapter, we have examined and illustrated the prospects for the use of subsidies to the differentiated goods sector of the economy financed by a proportional income tax. While the proportional income tax such as the one we used in this chapter potentially reduces utility due its effecting an increase in labour supply, a reduction in nominal income and a decline in output of the homogeneous good sector, we find that the resulting expansion of the differentiated goods sector in the form of a greater number of varieties for consumption or in an increased individual firm output increases utility such that a net gain in welfare is the final outcome.

A key take-away one can obtain from this chapter is that the type of subsidy policy we considered does not appear to yield negative welfare outcomes that have been identified as resulting from the more ‘conventional’ government consumption type of expenditure policy – typically due to crowding-out effects of private consumption. Thus, our results in this chapter complement the existing literature by showing that there is a possible second avenue via which welfare beneficial fiscal policy can be enacted. Instead of the more ‘conventional’ government consumption type of expenditure policy, a supply side tax-and-subsidy policy also appears to hold worthwhile promise to a welfare-maximising government as an alternative means to raise social welfare.

Appendix

A. Mathematical Derivations

A1. Deriving the Indirect Utility Function

Maximising the utility function,

$$U = \left(\frac{Y}{\mu}\right)^{\mu}\left(\frac{A}{1-\mu}\right)^{1-\mu} - \frac{L^{1+\gamma}}{1+\gamma},$$

subject to the budget constraint, we get the labour supply function as a function of the general price index, or

$$L^S = \left(\frac{w}{P_Y^{\mu}P_A^{1-\mu}}\right)^{\frac{1}{\gamma}}.$$

The demand functions for the Y- and A- goods are respectively

$$Y^D = \frac{\mu w L^S}{P_Y},$$

and

$$A^D = \frac{(1-\mu)wL^S}{P_A}.$$

Substituting the expressions for Y^D and A^D into U, we obtain an expression for the indirect utility function which we denote as V:

$$V = \left(\frac{wL^S}{P_Y}\right)^{\mu}\left(\frac{wL^S}{P_A}\right)^{1-\mu} - \frac{L^{1+\gamma}}{1+\gamma}.$$

L is the labour supplied by the representative consumer. Thus replacing L with L^S and substituting the expression for the labour supply function into V, we get

$$V = \left(\frac{w}{P_Y}\left(\frac{w}{P_Y^{\mu}P_A^{1-\mu}}\right)^{\frac{1}{\gamma}}\right)^{\mu}\left(\frac{w}{P_A}\left(\frac{w}{P_Y^{\mu}P_A^{1-\mu}}\right)^{\frac{1}{\gamma}}\right)^{1-\mu} - \frac{1}{1+\gamma}\left(\frac{w}{P_Y^{\mu}P_A^{1-\mu}}\right)^{\frac{1+\gamma}{\gamma}}$$

$$=\left(\frac{w}{P_Y^{\mu}P_A^{1-\mu}}\right)^{\frac{1+\gamma}{\gamma}}-\frac{1}{1+\gamma}\left(\frac{w}{P_Y^{\mu}P_A^{1-\mu}}\right)^{\frac{1+\gamma}{\gamma}}$$

$$=\frac{\gamma}{1+\gamma}\left(\frac{w}{P_Y^{\mu}P_A^{1-\mu}}\right)^{\frac{1+\gamma}{\gamma}}$$

which as mentioned in Section 3.4, is a function of real wage and the elasticity of labour supply.

B. Graphs

Figure 3-1: Equilibrium with the use of an Average Cost Subsidy

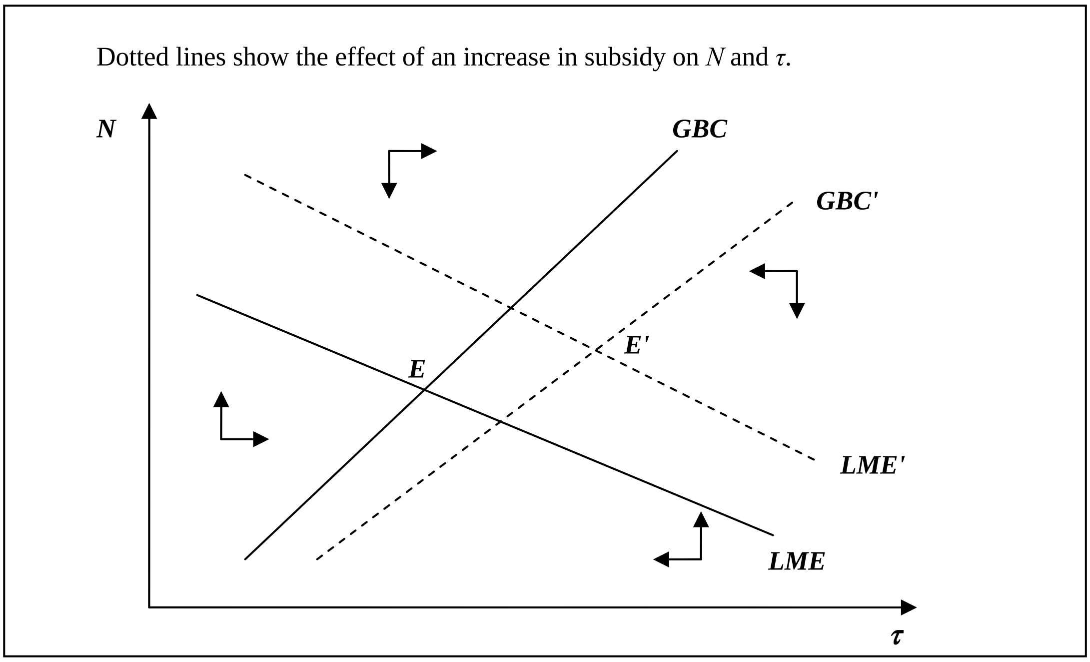


Figure 3-2: Tax-Subsidy Schedule with *ad hoc* values of *s*

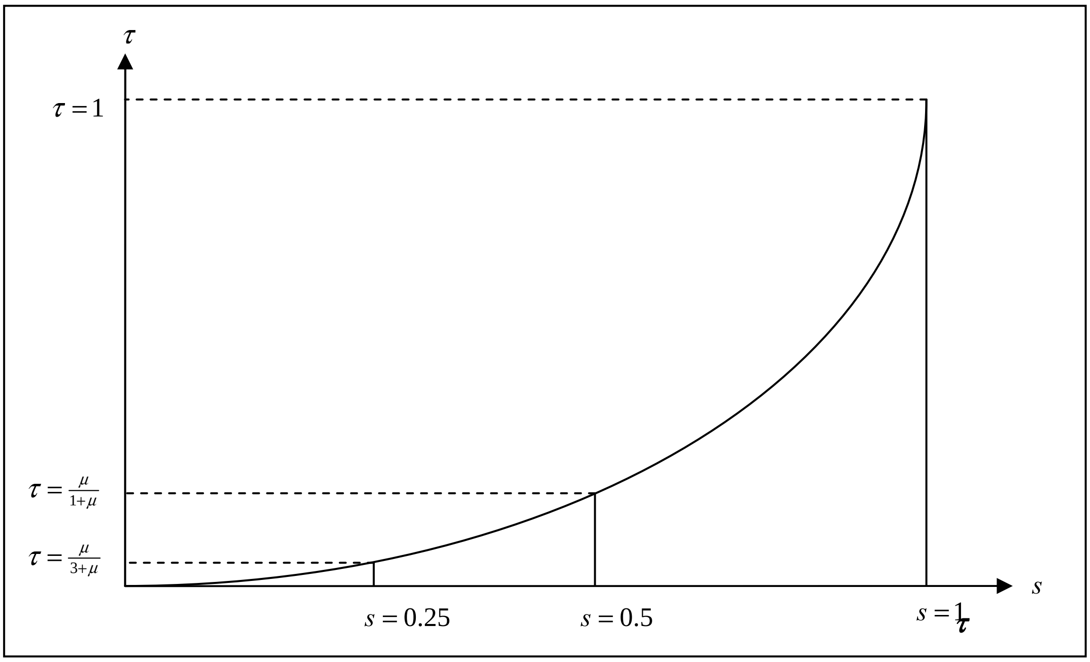


Figure 3-3: Equilibrium with the use of a Lump Sum Subsidy

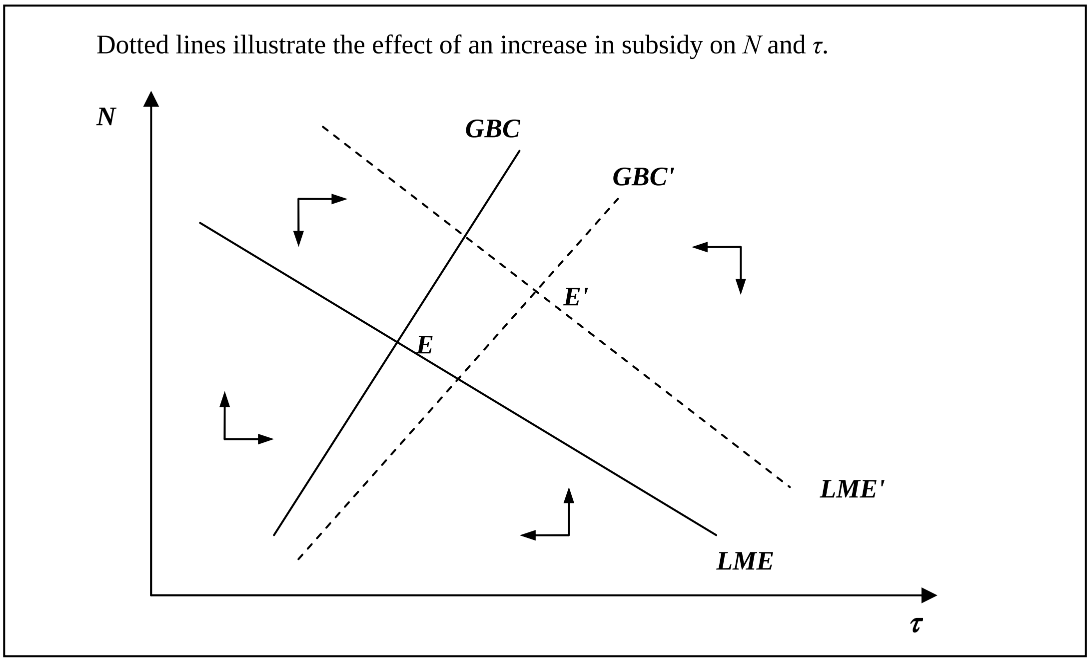
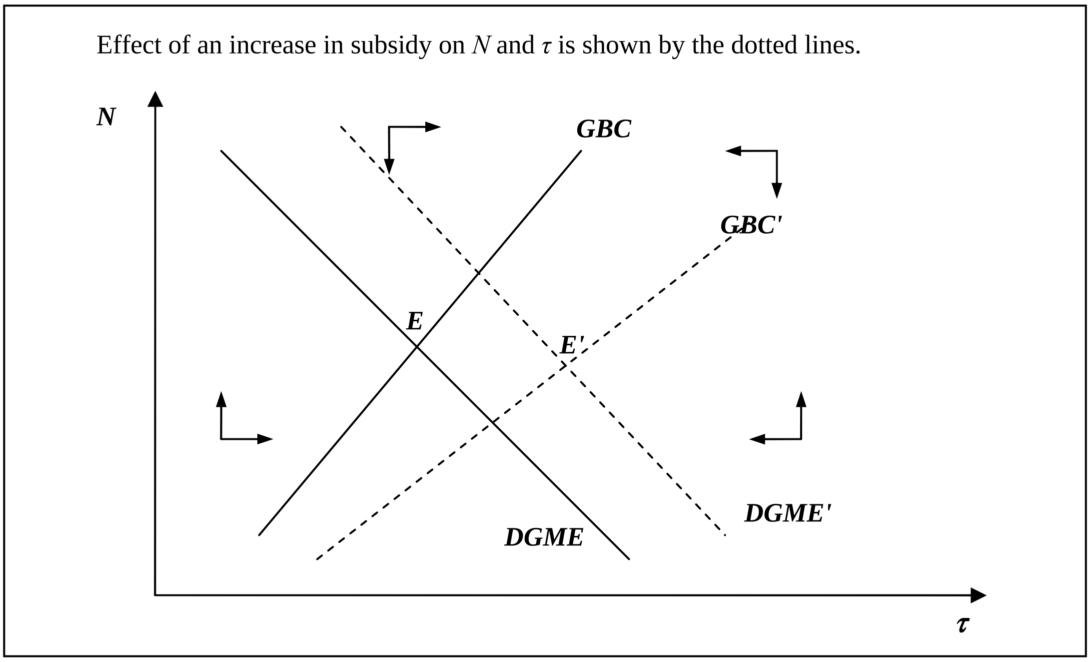


Figure 3-4: Equilibrium with the use of a Per Unit Subsidy



Chapter 4

Optimal Subsidies and Welfare in a Small Open Economy

4.1. Introduction

This chapter extends the benchmark model developed in Chapter 2 by including an external sector to the model. We treat labour supply as exogenous and examine the effects of a subsidy provision to firms in a small open economy. Much of the existing literature that analyses the welfare desirability of subsidy provisions, typically within the New Trade Theory context, has yielded ambiguous results. We show that this is not necessarily the case within a monopolistically competitive framework and that the use of a subsidy to average costs always leads to a Pareto improvement.

Incorporating imperfect competition into macroeconomics has yielded a bountiful harvest of insights on the desirability of government in the macro-economy. Such beneficial results are not just limited to the closed economy context, but also extend to the open-economy context [1]. Central to this theme is the assumption that firms experience increasing returns to scale in production and there are potential welfare gains to be garnered when a government uses fiscal expenditure to influence either demand or aggregate output. Matsuyama (1995) reviews and covers the theoretical underpinnings and related literature pertaining to this.

Assuming therefore, that firms have decreasing average costs of production,

[1] The New Open Economy Macroeconomics literature also utilises a similar framework, but the emphasis is on monetary and exchange rate policy issues. As in our previous chapters, our focus in here is on fiscal policy. See Corsetti (2007) for an overview of the former strand of the literature.

welfare improvements can be obtained if a benevolent and welfare-maximising government were to implement policies that raise demand for the firms' output. Defining fiscal policy broadly as any set of government tax or revenue-generating or expenditure policy, the strategic trade literature offers a rich area where this is examined in the open economy. The seminal contribution of Brander and Spencer (1985) is one such example. They illustrated how by providing a subsidy to an exporting firm, a government creates a rent shifting effect, or – more specifically – a result where profits from foreign firms are 'transferred' to domestic firms. Subsequently, output of the home firm is raised and there is a larger domestic industrial sector analogously speaking. The extra profits accruing to the country as a result of the subsidy policy more than offset the government subsidy bill, thus ensuing in an increase in welfare[2].

This welfare gain is not necessarily confined only to the policy-implementing country. As Mai and Hwang (1987) have shown, when a domestic government implements an export subsidy, welfare-improvements for the other country are also possible, so long as the country that implements the policy has either a larger international market share (in the case of a single exporting firm), or a larger number of domestic exporting firms relative to the foreign country. These results bring with them several suggestive implications. Firstly, if export subsidies and increased exports are socially beneficial, they ought to be used. And secondly, these results suggest (and this is supported by some empirical evidence) that higher export levels may result in greater economic growth (Greenaway and Kneller, 2005). Thus, there appears to be justification for export subsidies to be disbursed to firms.

However, an issue with the use of export subsidies relates to the robustness of

[2] Hamilton and Stiegert (2002) provide empirical evidence suggesting that this is an empirically valid result.

the findings. Brander and Spencer (1985) had used Cournot competition for their analysis. Under Bertrand competition, as Eaton and Grossman (1986) show, the optimal policy entails the use of a tax on exports as opposed to a subsidy! Markusen and Venables (1988) further highlight and illustrate this. Using a two-country model and considering a variety of settings, they find that the case for a policy intervention is not robust to varying assumptions concerning entry and market segmentation. For example, with free entry and exit of firms into each economy any policy intervention will reduce welfare – i.e. the socially optimal policy to pursue is free trade. However, if the number of firms is fixed no entry of firms is possible in each economy, an import tariff or export subsidy can be welfare improving. Similarly, the policy effects are greater when markets are segmented, e.g. due to trade frictions.

Despite the inconclusiveness of the economic literature, real world observations do seem to suggest that subsidies are provided in a more covert manner. While multinational international agreements tend to restrict governments from providing explicit export subsidies, support can, and is, provided in other ways[3]. Government expenditure in other forms of industry support such as government-backed finance schemes for firms, transport and infrastructure development, tax holidays, etc, that are actually permissible and may come under the generic term of ‘industrial policy’. Ishi (1999) for example, documents the industrial policy practices in Japan in the form of import restrictions, export promotion and government directives on resource allocation[4]. Apart from Japan, Cohen (2007) also provides a detailed historical overview of various industry-enhancing policies used by the French government. These are designed such that they protect and aid the growth of a number of specific industrial sectors against foreign competitors, thereby enhancing their development

[3] See the examples and references listed in Greenaway and Kneller (2005).
[4] Noland (1993) had previously examined the impact quantitatively and finds the use of such to be welfare improving.

and growth prospects.

4.1.1. Objective and Layout

While the exposition by Matsuyama (1995) outlines the theoretical underpinnings of the rationale for the use of government expenditure to raise aggregate demand and show how this can be welfare improving in the open-economy, Greenaway and Kneller (2005) hint that governments appear to also favour support policies for industries that influence aggregate supply. Such policies may take the form of export-competition, import-substitution or both. The outcome of using widespread domestic subsidies to support and influence local industrial output and growth in an open economy is perhaps somewhat more ambiguous.

Therefore, the objective of this chapter is to examine what possible outcomes arise if governments give economy-wide support to firms. We choose a monopolistically competitive market structure on which to build our model, with free entry and exit of firms into the market with little or no strategic interaction between them. We analyse the use of an average cost subsidy, which is in effect a wage subsidy. The welfare rankings of subsidy policies obtained from both Chapter 2 and Chapter 3 have shown clearly that this particular subsidy is welfare superior to either a per unit production subsidy or a subsidy to fixed costs. Hence, we take the government to be one who goes for the highest level of welfare and will therefore implement this policy.

Thus, this chapter tries to address the following specific questions. In an open economy with monopolistically competitive firms, does the provision of a subsidy yield a robust welfare improving outcome as in the case of the closed economy? Does a welfare maximising subsidy policy exist? What other possible effects are there in implementing support policies, or subsidies as we assume them to be, to firms within this market structure, e.g. on output and on the pattern of trade?

The rest of the chapter is structured as follows. Section 4.2 reviews the relevant related literature on the use of fiscal policy (in the broad sense as we categorised it earlier) in the open economy. We then lay out the basic model on which our subsequent analysis is built in Section 4.3. Section 4.4 proceeds to examine the outcomes of an average-cost subsidy policy implemented by the domestic government. We also spell out the rationale for this particular subsidy choice in this section. Section 4.5 concludes the chapter. A mathematical appendix contains the derivation of the model calibration followed by a listing of the graphs and some numerical results obtained from the analysis in Section 4.4.

4.2. Related Literature

4.2.1. Trade Instruments

There are several lines of analysis within the monopolistic competition literature concerned with how fiscal policy in the open economy potentially affects welfare. One of these involves influencing the demand for firms' output and studying the resulting welfare change.

One type of policy intervention, as explored by Venables (1982) and Lancaster (1991), consists of the imposition of tariffs on the imports of the differentiated good. These authors show that when imports are subject to a tariff, an expansion of the domestic differentiated good sector can result and give a welfare outcome that is superior to free trade. Venables (1982) and Lancaster (1991) had utilised a small open economy setting, where the domestic economy exports a homogeneous commodity and imports differentiated varieties. Hence, the tariff raises the price of imports such that it diverts domestic consumption away from differentiated imports in favour of domestic varieties. This subsequently induces the entry of new firms to the differentiated sector, thereby raising welfare. Jørgensen and Schröder (2005) however, study the use of tariff policies with a subsequent redistribution of the tariff revenue to consumers in a two-country scenario where only differentiated goods in a monopolistically competitive market structure are produced and traded. Using a specific tariff on the price of imports and an ad valorem tariff, they find that any tariff policy always yields a welfare outcome inferior to free trade, even with the tariff revenue being redistributed fully. In both tariff regimes, the tariff diverts domestic consumption away from importing varieties, leading to a reduction in the aggregate level of imports due either to a reduction in output of each exporting firm, or in the number of firms (varieties) traded. Hence, owing to consumers' love for variety, the resulting fewer number of varieties available for consumption causes social welfare to fall as compared to free trade[5].

An alternative instrument to the imposition of a tariff is the provision of export subsidies to firms in a monopolistically competitive market structure. Extending the Brander and Spencer (1985) analysis to monopolistic competition, Lin (1996) finds that consumers enjoy higher welfare levels as a result of a subsidy, as they end up with a greater aggregate level of consumption of the differentiated good. When both governments provide an export subsidy, prices of the differentiated imported varieties are lower in the respective importing countries. This diverts consumption away from the homogeneous good to the differentiated one. The higher demand for differentiated imports subsequently fuels an increase in the total number of differentiated varieties produced in both countries, generating a higher level of

[5] This possible conflict of outcomes is also illustrated by Sen (2005) who considers the scenarios of having either the differentiated or homogeneous sector to be non-traded. In the former, welfare is raised as tariffs induce the entry of firms into the sector. In the latter, import tariffs result in an exit of firms from the differentiated sector and leading to a welfare decline.

welfare to consumers with love-of-variety preferences.

4.2.2. Government Domestic Expenditure

The government's domestic expenditure can be used to influence demand or supply of output. In the former, the government can take on the role of a consumer and use its expenditure to raise aggregate demand of the economy, and expanding the increasing returns sector as a consequence. Examples of analysis along this line include those studied by Santoni (1999) and Andersen (2007). The focus of our interest here is in the latter, where government expenditure is used to influence costs and the level of output.

Of particular relevance to us here are the contributions of Flam and Helpman (1987) and Venables (1987), who examine the various effects of firm subsidies using monopolistic competition as the market structure. Flam and Helpman (1987) consider the case of a small open economy where a government utilises several different types of subsidy and tariff instruments. They find that welfare is raised from an increase in product variety when an import tariff is used, but varying subsidy tools appear to give no conclusive outcome on welfare. Within a two country context however, Venables (1987) finds that in the presence of individual firm market power, an import tariff or some form of cost subsidy always gives rise to higher welfare.

Along a similar vein, Bettendorf and Heijdra (2004) examine the impact of a production subsidy using an overlapping-generations model. They identify the possibility of a first-best outcome when a government adopts an import tariff and production subsidy policy combination, both instruments which were already found to give socially beneficial results individually. Their results go towards resolving the ambiguity of outcomes found by Flam and Helpman (1987), while lending support to the positive results obtained by Venables (1987).

The general implication one might get thus far is that government expenditure in the form of subsidies can have a positive effect on welfare[6]**.** Thus, with this in mind, we extend the existing literature by attempting to establish if there is an optimum subsidy for a small open economy that engages in trade of differentiated final goods with the rest of the world. We also study what outcomes arise from a widespread subsidy provision to firms in the increasing returns to scale sector. These include among others, the effects of the subsidy on firm output scale, the size of the sector and the pattern of trade.

The seminal contribution of Flam and Helpman (1987), which we use as the basis for our model, had assumed the differentiated sector of the domestic economy to be 'large', that the number of varieties produced domestically has an effect on the foreign price index of *Y*, thereby influencing real foreign expenditure. In contrast to them, we take the open economy to be 'small', such that both domestic price and the number of varieties produced have no effect on real expenditure of the rest of the world. As we will show in the next section, foreign demand and expenditure of the differentiated good is influenced only by price and the elasticity of substitution between each variety. The small open economy setting is interesting in that – as some have argued – globalisation means that each individual economy becomes smaller relative to the world as a whole. Furthermore, it is certainly the case that many newly industrialising economies that pursued trade liberalisation policies in recent decades are relatively small.

[6] However, as a word of caution, Francois (1992) finds that if the small open economy engages in trade in differentiated intermediates, subsidies do not yield any welfare gains and the optimal policy to adopt remains a non-interventionist one.

4.3. The Model

The base model employed here for our analysis follows from the specification spelt out in Flam and Helpman (1987). We take the case of a small open economy endowed with a labour force, L, which engages in trade with the rest of the world. Both the domestic economy and the rest of the world produce a homogeneous good, (A), and a differentiated commodity, (Y). To differentiate between them, the subscript F is added to denote foreign variables.

Specifically, all consumers are identical and consume both goods. A representative individual has the following Cobb-Douglas utility function,

$$U = \left(\frac{Y}{\mu}\right)^{\mu}\left(\frac{A}{1-\mu}\right)^{1-\mu}. \tag{4.1}$$

Income (I) is derived solely from the supply of labour, or $I = wL$. This is then fully expended on consumption, implying the budget constraint, $I = P_A A + P_Y Y$.

Y is a composite basket of goods following the Dixit and Stiglitz (1977) formulation, made up of a continuum of domestic varieties, y_i, and imports, x_{i_F}, where i is a specific variety from within the range $[0, N]$, with the corresponding foreign range $[0, N_F]$[7]. With trade, the composite basket Y takes the form:

$$Y = \left(\int_0^N y_i^{\frac{\sigma-1}{\sigma}}\,di + \int_0^{N_F} x_{i_F}^{\frac{\sigma-1}{\sigma}}\,di_F\right)^{\frac{\sigma}{\sigma-1}} \tag{4.2}$$

The corresponding price index can be found as:

$$P_Y = \left(\int_0^N p_i^{1-\sigma}\,di + \int_0^{N_F} p_{i_F}^{1-\sigma}\,di_F\right)^{\frac{1}{1-\sigma}}. \tag{4.3}$$

The individual's utility maximisation problem takes place in two stages. In the

[7] Note that while we have a continuum of domestic, N, and foreign, N_F, varieties, we will consider them as the 'number' of varieties available. Strictly speaking, this is incorrect, but it facilitates ease of intuitive understanding in the later stages.

first stage, maximising Eq. (4.1) with respect to the budget constraint yields the expenditure on each good, or

$$\mu I = P_Y Y \tag{4.4}$$

$$(1-\mu) I = P_A A \tag{4.5}$$

The second stage consists of allocating the expenditure share on Y, or Eq. (4.4), to each variety given the price index, P_Y, from Eq. (4.3). With each variety entering into the consumption function symmetrically, this gives the home demand function of each domestic i-th variety as:

$$y_i^D = \frac{\mu I}{P_Y}\left(\frac{p_i}{P_Y}\right)^{-\sigma} \tag{4.6}$$

and demand for each foreign variety:

$$x_{i_F}^D = \frac{\mu I}{P_Y}\left(\frac{p_{i_F}}{P_Y}\right)^{-\sigma}. \tag{4.7}$$

Given the assumption of a small open economy, domestic exports have no influence on the rest of the world. Therefore, trade flows from/to this small open economy do not affect the price and the number of the varieties produced in the rest of the world, or the world price index, P_{Y_F} . Assuming the elasticity of substitution between domestic exports to equal that between foreign varieties, the foreign demand function can be written as in Eq. (4.7). Since P_{Y_F} is exogenous, foreign demand of domestic varieties y is determined solely by p_i. Thus, denoting foreign expenditure with E_F (which is treated as exogenous), we can write the foreign demand function for domestically produced varieties without any loss of generality as:

$$y_i^{FD} = E_F p_i^{-\sigma}. \tag{4.8}$$

Finally, the domestic economy's indirect utility function is derived to be:

$$V = \frac{wL}{P_Y^{\mu} P_A^{1-\mu}}. \tag{4.9}$$

On the supply side, the homogeneous good is produced and traded under constant returns to scale with a unit labour requirement of unity. This implies a production function of the form:

$$A^S = L_A. \tag{4.10}$$

Firms in this sector pay a wage, *w*, to labour and in a perfectly competitive market with constant returns to scale, the profit function can be written as

$$\pi_A = P_A A^S - wL_A, \tag{4.11}$$

where the first term is the revenue a typical firm receives and the second term is its production costs.

For the differentiated sector, a typical firm uses labour as the only production input. In equilibrium, each firm's total output will equal the sum of home and foreign demand, or:

$$y_i^S = \left(y_i^D + y_i^{FD}\right) = \frac{\mu I}{P_Y}\left(\frac{p_i}{P_Y}\right)^{-\sigma} + E_F p_i^{-\sigma} \tag{4.12}$$

A fixed input of α is incurred in production with a marginal requirement of β for each unit of *y* such that the total labour demand by a typical firm *i* is:

$$l_i = \alpha + \beta y_i$$

The cost function of the firm is given as:

$$C_i = \left(\alpha + \beta y_i^S\right) w, \tag{4.13}$$

with a corresponding profit function of:

$$\pi_i = p_i\left(y_i^D + y_i^{FD}\right) - \left(\alpha + \beta y_i^S\right) w. \tag{4.14}$$

The firm maximises its profit subject to demand, and sells its output at a mark-up over its marginal cost of production, with price:

$$p_i = \frac{\sigma\beta w}{\sigma - 1}. \tag{4.15}$$

Substituting for p_i into the profit function, the optimal scale each firm operates at is found to be:

$$y_i^S = \left(y_i^D + y_i^{FD}\right) = \frac{\alpha(\sigma-1)}{\beta}. \tag{4.16}$$

With symmetry between firms, the subscript i denoting each firm can be dropped.

4.3.1. Normalisations and General Equilibrium

The homogeneous good is assumed to be produced under perfectly competitive conditions with a unit labour requirement of one and freely traded. Using this good as the numeraire implies:

$$P_A = 1, \tag{4.17}$$

from which follows that the wage rate is $w = 1$ in the A sector. With inter-sectoral labour mobility then results in wage equalisation between sectors, i.e. the wage in the Y-sector is thus also 1. Substituting $P_A = w = 1$ into Eq. (4.9), the indirect utility function can now be written as:

$$V = \frac{L}{P_Y^{\mu}} \tag{4.18}$$

As we have imposed a symmetric equilibrium, Eq. (4.3) can be written as

$$P_Y = \left(Np^{1-\sigma} + N_F p_F^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Thence, substituting Eq. (4.3) and Eq. (4.15) into the good market equilibrium condition for each variety in Eq. (4.12) and solving for N, we obtain the number of firms that exist in equilibrium:

$$N = \frac{\mu L}{\alpha\sigma - E_F\left(\sigma\beta/(\sigma-1)\right)^{1-\sigma}} - N_F\left(\frac{p_F(\sigma-1)}{\sigma\beta}\right)^{1-\sigma}. \tag{4.19}$$

The total labour requirement for the *Y*-sector is thus derived as

$$L_Y = Nl$$

$$= \alpha\sigma\left(\frac{\mu L}{\alpha\sigma - E_F\left(\sigma\beta/(\sigma-1)\right)^{1-\sigma}} - N_F\left(\frac{p_F(\sigma-1)}{\sigma\beta}\right)^{1-\sigma}\right). \tag{4.20}$$

Domestic labour allocation and supply of the homogenous sector is therefore:

$$L_A = L - Nl$$

$$= L - \alpha\sigma\left(\frac{\mu L}{\alpha\sigma - E_F\left(\sigma\beta/(\sigma-1)\right)^{1-\sigma}} - N_F\left(\frac{p_F(\sigma-1)}{\sigma\beta}\right)^{1-\sigma}\right)$$

$$= A^S. \tag{4.21}$$

Thence, the market clearing condition for the homogenous goods market implies that total imports of the *A*-good equals the difference between total expenditure and domestic supply, giving:

$$A^M = A^D - A^S$$

$$= \frac{\mu L E_F\left(\frac{\sigma\beta}{\sigma-1}\right)^{1-\sigma}}{\alpha\sigma - E_F\left(\frac{\sigma\beta}{\sigma-1}\right)^{1-\sigma}} - \alpha\sigma N_F\left(\frac{p_F(\sigma-1)}{\sigma\beta}\right)^{1-\sigma} \tag{4.22}$$

It is straightforward to verify too, that the trade balance condition holds, i.e.:

$$A^M + \left(N_F p_F x_F^D - Npy^{FD}\right) = 0. \tag{4.23}$$

The first term in Eq. (4.23) follows on from Eq. (4.22), which is the amount of the *A*-good imported[8] needed for the homogenous good market to clear. The first and second terms in parenthesis give the total value of the differentiated imports which are consumed domestically and that of the exported domestic varieties respectively. Thus, Eq. (4.23) equates the sum of the excess demands in both markets and ensures that both the *A*- and *Y*-good markets clear.

[8] The domestic economy can also be a net exporter of *A*, in which case A^M will be negative.

From the trade balance condition, a trade ratio (r), of the differentiated good can be defined as:

$$r = \frac{Npy^{FD}}{N_F p_F x_F^D}. \tag{4.24}$$

4.4. Firm Subsidies

4.4.1. Subsidy Provision

We analyse the effects of an average cost, or wage, subsidy financed by a proportional income tax, τ. A rationale for choosing such a subsidy policy is offered by Begg and Portes (1993) who find that relative to other types of subsidy policies, wage subsidies yield the greatest welfare gain[9]. We therefore assume that the domestic government grants a uniform subsidy, s, to all firms in the Y-sector for every unit of labour they employ. Proportional income tax is used both for simplicity, and because of the fact that such taxation generally constitutes a major component of government income worldwide. In the case of a small open economy, foreign expenditure is assumed as exogenous and unaffected by the domestic policy.

Recalling that $w = 1$, the wage subsidy alters the cost and profit functions of each firm as follows:

$$C_s = \left(\alpha + \beta y^S\right)(1 - s), \tag{4.25}$$

$$\pi_s = p\left(y^D + y^{FD}\right) - \left(\alpha + \beta y^S\right)(1 - s). \tag{4.26}$$

The price each firm sets for its own product is now

$$p_s = \frac{\sigma\beta(1 - s)}{\sigma - 1} \tag{4.27}$$

[9] Fleurbaey (1998) studies the resulting welfare effects using a general equilibrium model with monopolistic competition. He finds that when the government can either provide income subsidies or welfare payments, the former gives a larger welfare gain than the latter.

Inserting p_s into the profit function and imposing the zero-profit condition, it easy to verify that the optimal size of each firm remains unchanged as before, i.e.

$$y^S = \frac{\alpha(\sigma - 1)}{\beta}.$$

On the demand side, with a proportional tax, τ, imposed on the Home consumer and with the normalisation of $P_A = 1$, the budget constraint is redefined as:

$$(1-\tau)L = A + P_Y Y. \tag{4.28}$$

The solution to the representative consumer's optimisation problem, then results in the following allocation of expenditure over the two goods:

$$Y^D = \frac{(1-\tau)\mu L}{P_Y} \tag{4.29}$$

and

$$A^D = (1-\tau)(1-\mu)L\,, \tag{4.30}$$

respectively. The corresponding indirect utility function of the representative consumer becomes

$$V = \frac{(1-\tau)L}{P_Y^{\mu}}. \tag{4.31}$$

Taking foreign expenditure as constant, total demand for each variety is obtained to be:

$$y^D + y^{FD} = \frac{(1-\tau)\mu L}{P_Y}\left(\frac{p_s}{P_Y}\right)^{-\sigma} + E_F\, p_s^{-\sigma} \tag{4.32}$$

with the corresponding domestic demand for each imported variety as

$$x_F^D = \frac{(1-\tau)\mu L}{P_Y}\left(\frac{p_F}{P_Y}\right)^{-\sigma}. \tag{4.33}$$

The price index now becomes

$$P_Y = \left(N \left(\frac{\sigma\beta(1-s)}{\sigma-1} \right)^{1-\sigma} + N_F p_F^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{4.34}$$

The government's budget constraint is now given by:

$$\tau L = N_s sl$$

$$= N_s s\alpha\sigma. \tag{4.35}$$

From Eqs. (4.25) to (4.34), we can now restate the equilibrium conditions as follow. With the total labour demand of the differentiated sector given by $L_Y = Nl$ and utilising Eq. (4.25), total costs of production incurred by the sector is

$$NC_s = N(\alpha + \beta y)(1-s)$$

$$= N\alpha\sigma(1-s). \tag{4.36}$$

From the demand of each domestic Y-variety from Eq. (4.32), we can obtain the total revenue of firms in the Y-sector to be

$$Np_s\left(y^D + y^{FD}\right) = N\left(\frac{(1-\tau)\mu L}{P_Y} \left(\frac{p_s}{P_Y} \right)^{1-\sigma} + E_F p_s^{1-\sigma} \right). \tag{4.37}$$

Equating Eqs. (4.36) and (4.37) to impose the zero-profit condition, and substituting in all equilibrium variables except L and N, we can obtain an equilibrium condition that gives the labour demand from the differentiated sector at the zero profit equilibrium:

$$(1-\tau)\mu L - \left(\alpha\sigma(1-s) - E_F \left(\frac{\sigma\beta(1-s)}{\sigma-1} \right)^{1-\sigma} \right) \left(N_s + N_F \left(\frac{p_F(\sigma-1)}{\sigma\beta(1-s)} \right)^{1-\sigma} \right) = 0. \tag{4.38}$$

Using Eq. (4.38) and Eq. (4.10) into the aggregate labour market equilibrium condition, we obtain the equilibrium labour demand for the A-sector.

In the homogeneous good sector, similarly to the unsubsidised equilibrium case as given by Eq. (4.22), it is possible again that domestic supply may not meet domestic demand with imports of the A-good accounting for the difference between domestic A-good expenditure and supply. With free trade, the A-good market equilibrium implies that total domestic supply and imports meets demand, or

$$A^S + A^M = (1-\tau)(1-\mu)L. \tag{4.39}$$

For the differentiated goods market, we obtain the market clearing condition from Eq. (4.16) and (4.32) which gives

$$\frac{\alpha(\sigma-1)}{\beta} = \frac{(1-\tau)\mu L}{P_Y}\left(\frac{p_s}{P_Y}\right)^{-\sigma} + E_F p_s^{-\sigma}$$

$$= \left(\frac{(1-\tau)\mu L}{\left(N_s\left(\sigma\beta(1-s)/(\sigma-1)\right)^{1-\sigma} + N_F p_F^{1-\sigma}\right)} + E_F\right)\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{-\sigma}. \tag{4.40}$$

The government budget constraint is given by:

$$\tau L = N_s s\alpha\sigma.$$

The final equilibrium condition is given by the trade balance[10]:

$$A^M + \left(N_F p_F x_F^D - N_s p_s x^D\right) = 0\,. \tag{4.41}$$

We take s to be exogenous and assume that the government adjusts the tax rate to balance its budget. This reduces the endogenously determined variables to τ and N_s. We first examine qualitatively the existence and stability of the equilibrium in the $[\tau, N]$ space. Denoting the differentiated goods market equilibrium as *DGME* and the government budget constraint as *GBC* respectively, we approximate them to straight lines, and depict them as shown in Figure 4-1. The locus of each line shows the combinations of τ and N that satisfy the labour market equilibrium condition and the government budget constraints.

Beginning at any arbitrary point on the government budget constraint, consider a

[10] It can be checked that any combination of two equilibrium conditions can be used to obtain the rest. As an example, in Appendix A1, the government budget constraint can be obtained using the trade balance and the labour market equilibrium.

ceteris paribus increase in *N*. This moves us to a new point on the *GBC* and increases government expenditure. The government will therefore have to raise the tax rate in order to finance the increase in subsidy, or *N* needs to fall in order for the government budget constraint to be satisfied, with the movement paths as shown by the horizontal arrows. Similarly, for the differentiated goods market equilibrium, at all points on *DGME* the differentiated good market clear – with the vertical arrows showing the movement paths above and below the locus. A *ceteris paribus* increase in τ takes us above the curve as it lowers disposable income and demand. This reduces profits and leads to exit of firms, hence *N* will fall and this reduces supply until the *DGME* is restored. The opposite holds below *DGME*. Hence, this curve slopes downwards.

The equilibrium will be stable as long as the *GBC* is steeper than the *DGME*, such that starting from any arbitrary point between the two loci, the economy will converge to *E* as shown in Figure 4-1. The effect of a change (in this case, an increase) in the subsidy, *s*, can be examined qualitatively. This is indicated by a shift in both the *GBC* and the *DGME* as depicted by the dotted lines *GBC′* and *DGME′* that results in a higher *N* and τ with a new stable equilibrium point, *E′*. Using the differentiated goods market clearing condition and the government budget constraint, we can solve for the *ad hoc* (i.e. not optimal) general equilibrium values of τ and N_s in terms of the other exogenous variables. These are:

$$\tau = \frac{\alpha s \sigma}{L}\left(\frac{\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)}{\alpha\sigma(1-s(1-\mu)) - E_F\left(\dfrac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}}\right) \tag{4.42}$$

and

$$N_s = \frac{\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)}{\alpha\sigma(1-s(1-\mu)) - E_F\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}}. \tag{4.43}$$

Analysing the *ad hoc* solutions for τ and N_s gives some interesting insights. Firstly, Eq. (4.43) shows that the use of a positive subsidy, accompanied by the associated positive tax, results in an increase in the number of firms in the *Y*-sector[11]. Specifically, differentiating Eq. (4.43) with respect to s yields:

$$\frac{\partial N_s}{\partial s} = \left(\frac{\left(\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)\right)}{\left(\alpha\sigma(1-s(1-\mu)) - E_F\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}\right)} \right.$$

$$\left. \frac{\left(\alpha\sigma(1-\mu) + E_F\sigma\beta\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{-\sigma}\right)}{\left(\alpha\sigma(1-s(1-\mu)) - E_F\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}\right)}\right)$$

$$+\frac{\alpha\sigma^2 N_F p_F^{1-\sigma}\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma}}{\alpha\sigma(1-s(1-\mu)) - E_F\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}}. \tag{4.44}$$

Note that all the terms in Eq. (4.44) are positive: it therefore follows that $\frac{\partial N_s}{\partial s} > 0$, i.e. the total number of firms in the *Y*-sector is larger as a result of the subsidy. As each firm's output remains unchanged post-subsidy, the larger number of firms, N_s thus translates into an unambiguously higher total output, *Y*. These results are similar to both Flam and Helpman (1987) and Bettendorf and Heijdra (2004) who consider the

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[11] Notice that the fraction comprised of the two terms in parenthesis in Eq. (4.42) is essentially the expression for N_s in Eq. (4.43). With $N_s > 0$, it follows on that the *ad hoc* solution for τ is positive as α, s and σ all take positive values.

use of output or export subsidies. Secondly, given consumers' love of variety, the larger number of varieties (firms) also suggests the possibility of the subsidy resulting in a higher level of social welfare[12]. Should this be the case, then an optimal policy tax-subsidy rule ought to exist.

4.4.2. Welfare

Previous contributions as mentioned in Section 4.2 seem to suggest ambiguous outcomes on the benefits of subsidy policies on a whole. Our intuition, however, points to a possible welfare increase as a result of the subsidy which we now examine. Therefore, using the *ad hoc* solutions for τ and N_s and making the other relevant substitutions into the indirect utility from Eq. (4.31), we get:

$$V_s = \frac{\mu L\left(1 - \dfrac{\alpha s\sigma\left(\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)\right)}{L\left(\alpha\sigma(1-s(1-\mu)) - E_F\left(\dfrac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}\right)}\right)}{\left(\dfrac{\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)}{\alpha\sigma(1-s(1-\mu))\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F} + N_F p_F^{1-\sigma}\right)^{\frac{\mu}{1-\sigma}}} \tag{4.45}$$

Differentiating with respect to s and evaluating at $s = 0$, the change in V gives

$$\left.\frac{\partial V_s}{\partial s}\right|_{s=0} = (\mu L)^{\frac{\mu}{\sigma-1}} \frac{\alpha\sigma}{\sigma-1}\left(\frac{\sigma-1}{\sigma\beta}\right)^{1-\sigma}\left(\alpha\sigma\left(\frac{\sigma-1}{\sigma\beta}\right)^{1-\sigma} - E_F\right)^{\frac{\sigma-1+\mu}{1-\sigma}}$$

$$\left(\mu(1-\mu)L + (\sigma-1+\mu)N_F p_F^{1-\sigma}\left(\alpha\sigma\left(\frac{\sigma-1}{\sigma\beta}\right)^{1-\sigma} - E_F\right)\right) > 0. \tag{4.46}$$

Hence, starting with a zero subsidy, a small increase in subsidy unambiguously

[12] Recall the 'love-of-variety' aspect from the indirect utility function, V, where a larger N translates to a higher level of welfare.

increases welfare, with the driving force behind this effect stemming from the larger number of varieties now available for consumption. This stands, in particular, against Flam and Helpman (1987) who identify the possibility of a welfare *loss* when an output or export subsidy is given to *Y*-firms. In their model, while both subsidies induce new entry of firms into the domestic differentiated-good sector, the reduction in price of the *Y*-good potentially lowers the foreign price index of *Y*, leading to an increase in demand for domestic exports. Thus, while the subsidy leads to an increase in N which raises welfare, the fall in domestic consumption can override any gains such that the outcome is a net deterioration in welfare.

In our case however, no such effect occurs from the subsidising of average costs, and this is due to our small open economy assumption which implies that real foreign expenditure is independent of N and any change in domestic consumption stems solely from a change in disposable income. Therefore, despite the fact that the higher level of taxation necessary to finance the policy lowers disposable income, the utility gain from the increase in varieties more than compensates for that lost from a reduction in the quantity of total consumption. Having established this, we now examine whether an optimal policy rule exists.

4.4.2.1. Benchmark Calibration

Owing to the complexity of the algebra involved, we illustrate the presence of an optimal policy using a series of numerical simulations. To determine the existence of an optimal policy rule, we make use of the initial equilibrium conditions together with the following parameter values of $\alpha = 0.01$, $\beta = 0.05$, $\mu = 2/3$, $N = 5$ and $N_F = 15$ to solve for the initial values of the other variables.

For the elasticity of substitution, we follow the results found by Broda and Weinstein (2006). Using American data between 1970 and 2000, they estimated a series of elasticities of substitution between various goods. They find that on average, the value of σ has decreased from 8 to 3 over the entire time period. Therefore, we use an average of their estimations, or $\sigma = 5$[13]. The value of the subsidy, s, lies in the range of $[0,1]$. Figure 4-2 shows the welfare effect when the subsidy is implemented.

The shape of the utility function is concave. When a small subsidy is given, welfare first increases with the level of subsidy. However, this increase is not continuous as can be seen from Figure 4-2. Utility increases until it reaches a peak, beyond which it subsequently declines. The presence of such a peak indicates the existence of a welfare maximising subsidy rule[14]. The intuition behind this result is straightforward. When *Y*-sector firms are accorded a subsidy from the government, new firms enter the market as production costs are now lower. This introduces the availability of more varieties of the differentiated good for consumption. As consumers love variety, the increase in the number of varieties gives an increase in welfare. However, recall that consumers are taxed in order to finance the government's subsidy bill. This in turn lowers disposable income, reducing the quantity consumed of both *Y* and *A* goods. Lower quantities of consumption contribute instead to lower welfare and work towards an offset of the welfare effect of a wider variety. Thus, to the left of the maximum in Figure 2, the increase in welfare from greater variety overrides the welfare loss from a reduction in consumption. Beyond the maximum, the benefit gained from consuming an additional variety of the *Y*-good is less than what is lost from the reduction in quantity consumed. The value of s corresponding to the peak is therefore the value

[13] See Appendix A2 for calibration.

[14] The qualitative nature of the result holds for a range of values for β. See the numerical results in Table C-1.

of the subsidy which gives the highest level of social welfare.

One noteworthy point of our result here is that it runs contra to that of Flam and Helpman (1987). They had established that the use of a tariff contributes to a positive terms-of-trade effect and subsequently raises welfare[15]. Using an export or output subsidy however, leads to a negative terms-of-trade effect and gives an ambiguous welfare outcome at best. Our choice of an average-cost subsidy similarly gives rise to a negative terms-of-trade effect[16]. Thus, coupled with the results from the tariff imposition, one could plausibly expect a resulting fall in welfare.

Instead, this does not happen in our case. As N_F is exogenous, the strictly positive gain in welfare starting from a small positive deviation from $s = 0$ is driven by the reallocation of resources which expands the Y-sector in both variety provision and aggregate output[17]. This subsequently corrects a distortion of 'too few' or 'too little' varieties and output in the monopolistically competitive sector. Thence, even with a deterioration in the terms-of-trade in our case, the subsidy helps correct this distortion and serves to generate a positive change in welfare.

4.4.2.2. Size Effect

A concern against the recommendation of any form of tax or subsidy policy in the presence of increasing returns is the lack of robustness in results. Apart from the results obtained by Markusen and Venables (1988), Klette (1994) further argued that another determining factor which affects the choice of the optimal policy rule in a small open economy relates to the size of the domestic differentiated sector relative to the rest of the world and the number of firms. He considers the case of an export

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[15] This is because the tariff induces new entry into the domestic differentiated sector, enlarging the scope of variety available for consumption, raising total output or both.
[16] Recall that there is a lowering of prices as shown from Eq. (4.27).
[17] As the output per firm remains unchanged post-subsidy, the positive change in N as found from Eq. (4.44) translates to an increase in aggregate output of Y.

subsidy provision by the home government and finds that when an economy becomes smaller relative to the rest-of-the-world, the welfare-improving policy can potentially switch from an export tax to an export subsidy.

Thus, while we found the existence of an optimal subsidy level in our benchmark calibration, it is interesting to verify the robustness of our result to see if the relative size of the economy with respect to the rest of the world may potentially change the optimal policy rule. Using the same parameterisation values for α, β and μ, Figure 4-3 shows the welfare levels for a larger N_F, e.g. $N_F = 50$ [18].

It is clear that the main qualitative result of the existence of an optimal subsidy holds regardless of the value of N_F. However, as N_F gets larger (i.e. the small open economy becomes progressively *smaller*) the size of the optimal subsidy falls. In the limiting case of $N_F \to \infty$, we get $s \to 0$, i.e. no intervention is the optimal policy. Thus, so long as a country is small such that $N_F \neq \infty$, there are still gains to welfare to be realised from subsidising the increasing returns to scale sector.

4.4.2.3. No Initial Trade in *A*-good

We also examine the case when there is no initial trade involved in the *A*-good, which implies that the market clearing condition for A is $A^S = A^D$ and that $L_A = (1-\mu)L$. Keeping the other parameters fixed, we thus solve for μ to obtain $\mu = 0.25$ [19]. As before, we then examine the welfare effect with the use of a subsidy within the range of $[0,1]$. The results are shown in Figure 4-4.

Even in this case, the implementation of a tax and subsidy policy combination raises welfare and there exists an optimal tax-subsidy rule.

[18] To put this in context, consider small economies such as Singapore and Hong Kong in this case and economies such as Canada and Malaysia for the benchmark case.
[19] See Appendix A2.

4.4.3. Comparative Statics

4.4.3.1. Effects to Labour Allocation and Output

We now turn to examine the effect on output and labour allocation between sectors as a result of the subsidy provision. Using the *ad hoc* solutions for τ and N_s, total labour demand in the differentiated good sector can be obtained. This is now

$$L_Y = N_s l$$

$$= \frac{\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)}{\alpha\sigma(1-s(1-\mu)) - E_F\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}}(\alpha+\beta y)$$

$$= \frac{\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)}{(1-s(1-\mu)) - \frac{E_F}{\alpha\sigma}\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}}. \tag{4.47}$$

Differentiating with respect to s and evaluating gives

$$\frac{\partial L_Y}{\partial s} = \frac{N_F p_F^{1-\sigma}\left(\alpha\sigma^2(1-s)\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma}\right)}{(1-s(1-\mu)) - \frac{E_F}{\alpha\sigma}\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}}$$

$$+\left(\frac{\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)}{\alpha\sigma(1-s(1-\mu)) - E_F\left(\frac{\sigma\beta(1-s)}{\sigma-1}\right)^{1-\sigma}}\right.$$

$$\left.\frac{\alpha\sigma(1-\mu) + E_F\sigma\beta\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{-\sigma}}{(1-s(1-\mu)) - \frac{E_F}{\alpha\sigma}\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma}}\right) > 0, \tag{4.48}$$

which implies that total labour demand in the differentiated goods sector is now higher. With the initial assumption of a perfectly inelastic labour supply, reallocation of labour has to take place from the *A*-good sector to the *Y*-sector such that total output and, thence, imports of the *A*-good must increase correspondingly.

Thus, a subsidy to firms in the *Y*-sector results in an increase in aggregate output of *Y* and simultaneously leads a reduction in output of the *A*-good.

4.4.3.2. Trade Effects

We can also determine the effects of the subsidy provision on the pattern of trade. Using the *ad hoc* solutions for τ and N_s with the expressions for P_Y, y^{FD} and x_F^D into the differentiated goods trade ratio, *r*, the post-subsidy trade ratio now becomes

$$r = \left(\frac{N_s}{N_F}\right)\left(\frac{p_s}{p_F}\right)^{1-\sigma} \frac{E_F\left(N_s p_s^{1-\sigma} + N_F p_F^{1-\sigma}\right)}{(1-\tau)\mu L} \tag{4.49}$$

The larger N_s and a smaller denominator in suggest that the value of *r* is now greater than that in Eq. (4.24). However, p_s is now also smaller as a result of the subsidy. To resolve the ambiguity, we substitute for p_s, N_s and τ, into Eq. (4.49) and differentiate with respect to *s* to obtain:

$$\frac{\partial r_s}{\partial s} = \frac{E_F \alpha\sigma\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma}}{\left(\alpha\sigma(1-s(1-\mu))\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)\left(\alpha\sigma(1-s)\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)}$$

$$\left(\frac{\left(\mu L - N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s)\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)\right)\left(\dfrac{\sigma(1-s(1-\mu))-\mu}{1-s}\right)}{N_F p_F^{1-\sigma}\left(\alpha\sigma(1-s(1-\mu))\left(\dfrac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma} - E_F\right)} \right.$$

$$+\frac{\mu\sigma L}{N_F p_F^{1-\sigma}}\left(\alpha\sigma(1-s)\left(\frac{\sigma-1}{\sigma\beta(1-s)}\right)^{1-\sigma}-E_F\right)^{-1}\Bigg)>0 \tag{4.50}$$

Thus, exports of the differentiated good have increased as a result of the subsidy provision.

The underlying intuition for this can be explained as follows. A subsidy reduces the price of each variety of Home's *y*-goods, and raises total demand for an individual firm's product. This subsequently induces entry into the sector, thereby expanding the number of firms in equilibrium. With output per firm unchanged, Npx^D is now larger on aggregate. A secondary effect of the subsidy comes from the imposition of the proportional income tax to finance it. As p_F is unchanged, a lower income of $I=(1-\tau)L$ as a result of the tax simultaneously reduces domestic demand for imported varieties such that the value of $N_F p_F x_F^D$ is now smaller. In turn, the ratio r is now larger as compared to the pre-subsidy value. Thus a subsidy policy in this context amounts to having a concurrent export promotion and import substitution effect to the *Y*-good sector.

We can thus identify the effect of the subsidy on the pattern of trade by analysing the impact of s on r. A value of $r=1$ implies that the domestic economy exports as much differentiated goods as it imports. If $r>1$, the domestic economy is a net exporter of *Y*-goods and a net importer of *A*. The reverse holds for $r<1$. Using the parameter values from the benchmark calibration in Section 4.4.2.1, Figure 4-5 shows the values of r for a range of s. Thus, the domestic economy is initially a net importer of differentiated goods. With the subsidy provision, its pattern of trade changes gradually until it finally becomes a net exporter of *Y*. The case where there is initially no trade in *A* is also examined, giving an identical result as shown in

Figure 4-6. With the provision of the subsidy, the domestic economy again becomes a net exporter of Y and a net importer of A.

Our results thus far in Section 4.4.3 suggest the following. Industrial policy in the form of industry subsidies not only raises the aggregate output of the monopolistically competitive sector, it also leads to a shift in the pattern of trade away from the A-good to the country becoming a net exporter of the increasing returns to scale Y-good. Furthermore, corroborating this are the results from Section 4.4.2 indicating there are welfare gains from the subsidy provision. This could make such a policy instrument attractive to implement for a country looking at some form of import substitution or export promotion of the increasing returns to scale sector.

4.5. Conclusion

In summary, we explored the plausibility for the use of a tax-subsidy combination as a policy tool in the context of a small open economy in this chapter. Our primary results indicate that such a policy potentially leads to an improvement in welfare with the main driver being an increase in the number of varieties of Y-goods available for consumption. We also find the presence of the existence of an optimal subsidy level which is welfare maximising. Our results are consistent within a range of plausible parameter values which take into account different cost levels, expenditure shares and the size of the domestic economy. Thus, our findings also help towards resolving some of the ambiguity present within the existing economic literature.

Besides affecting the level of welfare, the use of subsidies as a policy instrument also gives rise to other effects in the economy. Subsidies to firms in the increasing returns to scale sector potentially results in a reallocation of labour in favour of the differentiated sector, thereby leading to a corresponding contraction of output (and size) of the homogeneous sector. Another finding is the shift in exports of the economy in favour of the differentiated good, leading to an inflow of the homogeneous good. The underlying mechanism driving this stems from the subsidy creating a concurrent import-substitution and export-promoting type of outcome.

From a more practical perspective, our results appear to point in general to the validity of use of policies which favour the industrial sectors of the economy by governments. The crux of the argument here is that the presence of imperfectly competitive markets, such as monopolistic competition as used here, generates socially sub-optimal welfare outcomes. This creates a rationale for the use of government intervention to correct for this welfare sub-optimality. The use of a subsidy to average costs, or analogously wages, financed by the imposition of an income tax corrects this sub-optimality, resulting in a greater level of social welfare. This then can offer some theoretical explanation for the experience of countries such as Japan and France, as mentioned at the start of this chapter, which have used industry support policies.

Appendix

A. Mathematical Derivations

A1. Deriving the Government Budget Constraint

We can check that Walras' Law holds by using the trade balance and the labour market equilibrium condition to obtain the government's budget constraint.

We first rewrite the trade balance from Eq. (4.23) as:

$$\left(L_A-(1-\mu)(1-\tau)L\right)+\left(N_s p_s\left(y^S-y^D\right)-N_F p_F x_F^D\right)=0$$

This can be rearranged to yield:

$$\left(L_A-(1-\mu)(1-\tau)L\right)+\left(N_s p_s y^S-\left(N_s p_s y^D+N_F p_F x_F^D\right)\right)=0\,,$$

where $N_s p_s y^S$ is the total revenue in the Y-sector and the second term in brackets within the same parenthesis is total domestic expenditure on the differentiated good. The zero-profit condition requires total revenue to equal total costs. Total costs is the total wage bill incurred by all firms in the sector, $Nl(1-s)$, while total expenditure is given by the demand function from Eq. (4.29) multiplied by the price index, or $(1-\tau)\mu L$. Substituting into the trade balance, this becomes:

$$\left(L_A-(1-\mu)(1-\tau)L\right)+\left(N_s l(1-s)-(1-\tau)\mu L\right)=0\,.$$

Using the labour market equilibrium of $L=L_A+N_s l$ and substituting into the trade balance, this simplifies to give the government budget constraint, i.e.:

$$\tau L=N_s sl\,.$$

A2. Calibration

This section lays out the equations and solutions used to calibrate the model for the numerical simulations presented in Section 4.4.4. To calibrate the model, we make use of the initial equilibrium set out in Section 4.3.1 with the following assumptions. In the context of a small open economy, N_F, p_F and E_F are taken as exogenous. Also, as labour supply is exogenous, the initial values found for the parameter L is thus also taken as fixed.

Using the expression for N in Eq. (4.19), we can solve for E_F in terms of α, β, μ, σ, L, p_F, N and N_F. This gives

$$E_F = \frac{N\alpha\sigma - \mu L + N_F\left(\dfrac{p_F(\sigma-1)}{\sigma\beta}\right)^{1-\sigma}}{N\left(\dfrac{\sigma\beta}{\sigma-1}\right)^{1-\sigma} + N_F p_F^{1-\sigma}}$$

Substituting the values for each variable will yield the initial value for E_F. We assume further that $p_F = p = \frac{\sigma\beta}{\sigma-1}$. Initial values of both E_F and p_F can be obtained by substituting values for N, N_F, α, β, μ and σ.

Numerical values for p_F and E_F can now be obtained using a set of plausible parameter values. For a benchmark, the values used for the calibration are $\alpha = 0.01$, $\beta = 0.05$, $\mu = 2/3$, $L = 1$, $N = 5$ and $N_F = 15$ [20]. The value of the elasticity of substitution follows on from the average of the values obtained by Broda and Weinstein (2006), or $\sigma = 5$. The second simulation uses the same values for N, α, β, μ and σ, but N_F is increased in this case to 50.

---

[20] The choice of $N_F = 15$, can be justified on the grounds that the concept of 'small' is relative. In this sense, we can take 5 to be small relative to 15. Using $N_F = 50$ for the second case examines how 'size', i.e. how small the economy is relative to the rest of the world, affects the optimum subsidy. What we mean by small is that the economy cannot affect the world market size and the aggregate price index of the differentiated good.

Finally, substituting the found numerical values of p_F and E_F together with the values for α, β, and μ, N, N_F and L into V and evaluating the welfare levels for the range $s = [0,1]$ yields each of the results as illustrated in Figures 2-3. The numerical values of these solutions are found in Table C-1 in Appendix C.

In the final case where there is no trade in the A-good, the value of μ is solved to find the expenditure share in which no trade in the A-good takes place initially, or the conditions that

$$A^S - A^D = 0$$

and

$$Npy^{FD} - N_F p_F x_F^D = 0$$

hold. This implies

$$L_A - (1-\mu)L = 0$$

and

$$\mu L = Nl \, .$$

Substituting for $l = \alpha + \beta y$ and y and solving yields

$$\mu = \frac{N\alpha\sigma}{L} \, .$$

Using the values of $N = 5$, $\alpha = 0.01$ and $\sigma = 5$, we get

$$\mu = 0.25.$$

The initial and post-subsidy values of V are reported in Table C-2 in Appendix C.

B. Graphs

Figure 4-1: Equilibrium of N and τ with an Average Cost Subsidy

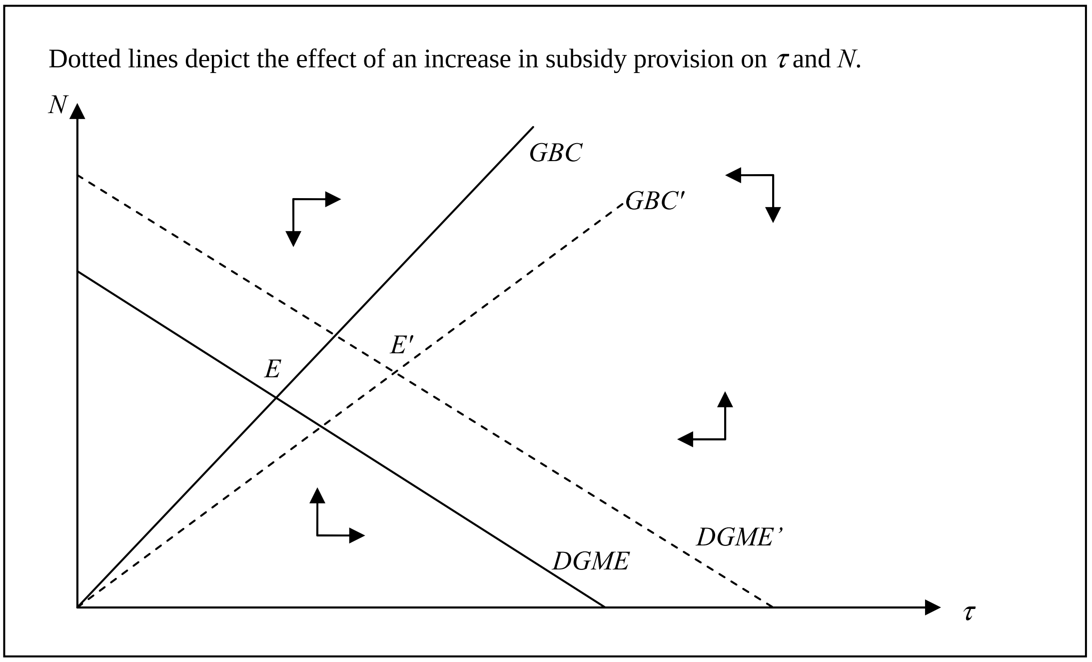


Figure 4-2: Benchmark Calibration

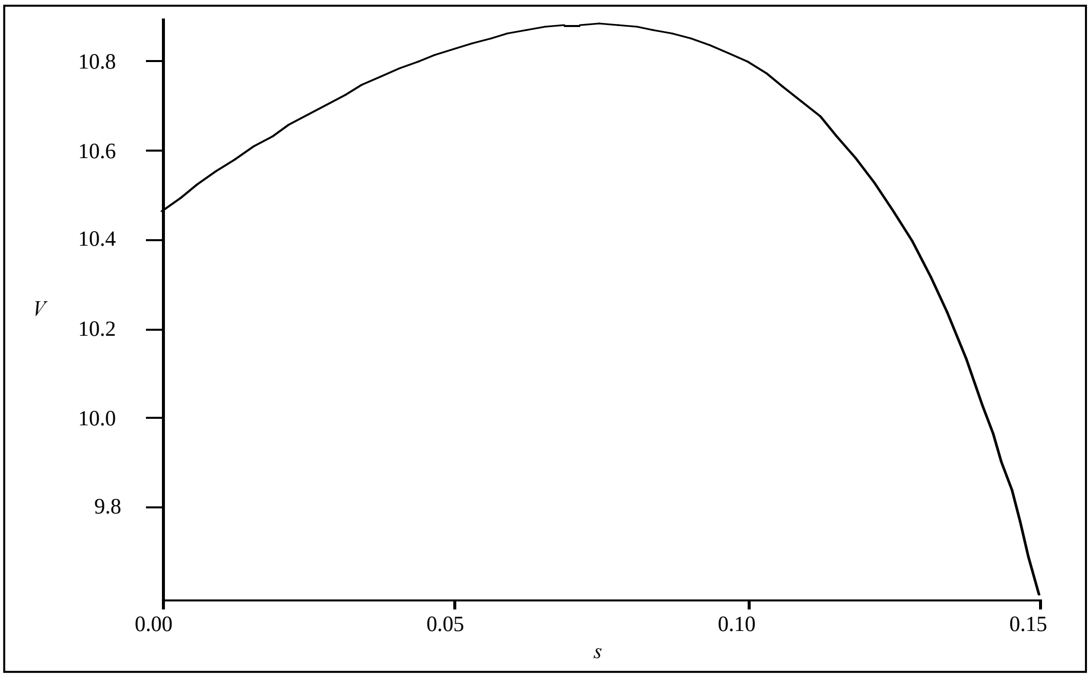


Figure 4-3: Subsidies and Welfare when $N_F = 50$

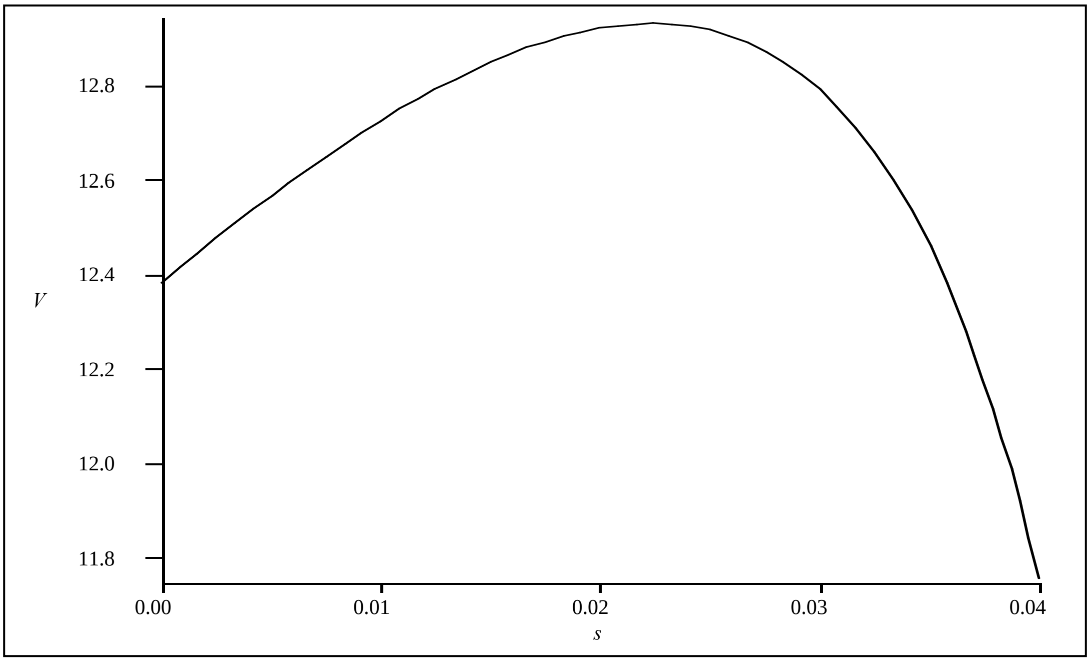


Figure 4-4: Subsidy and Welfare levels with no Initial *A*-good Trade

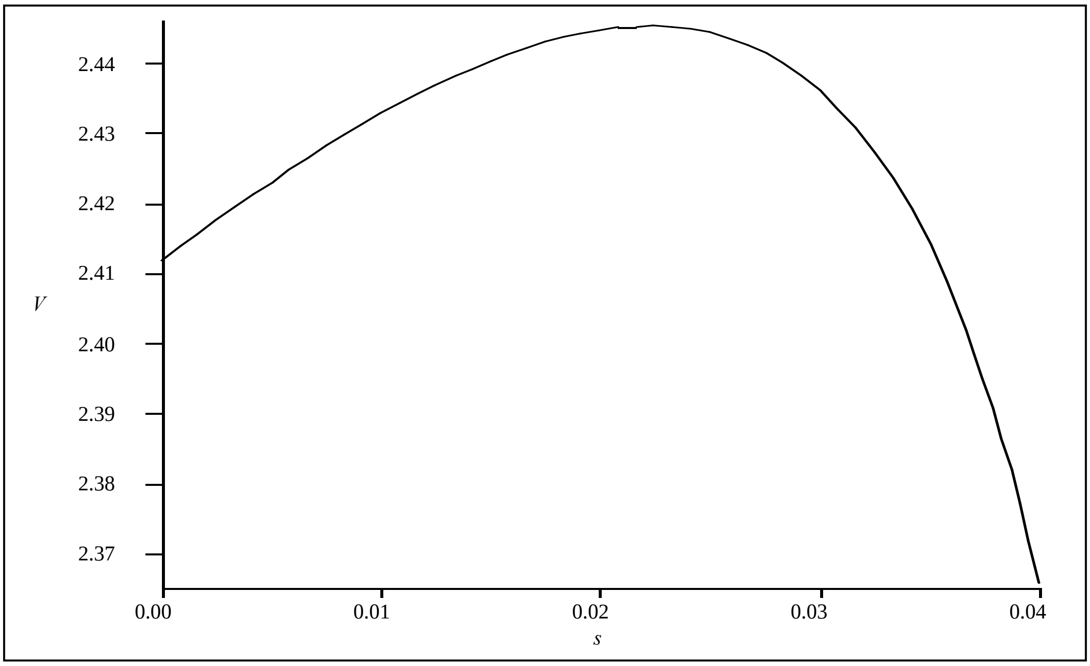


Figure 4-5: *Y*-good Trade Ratio in the Benchmark Case.

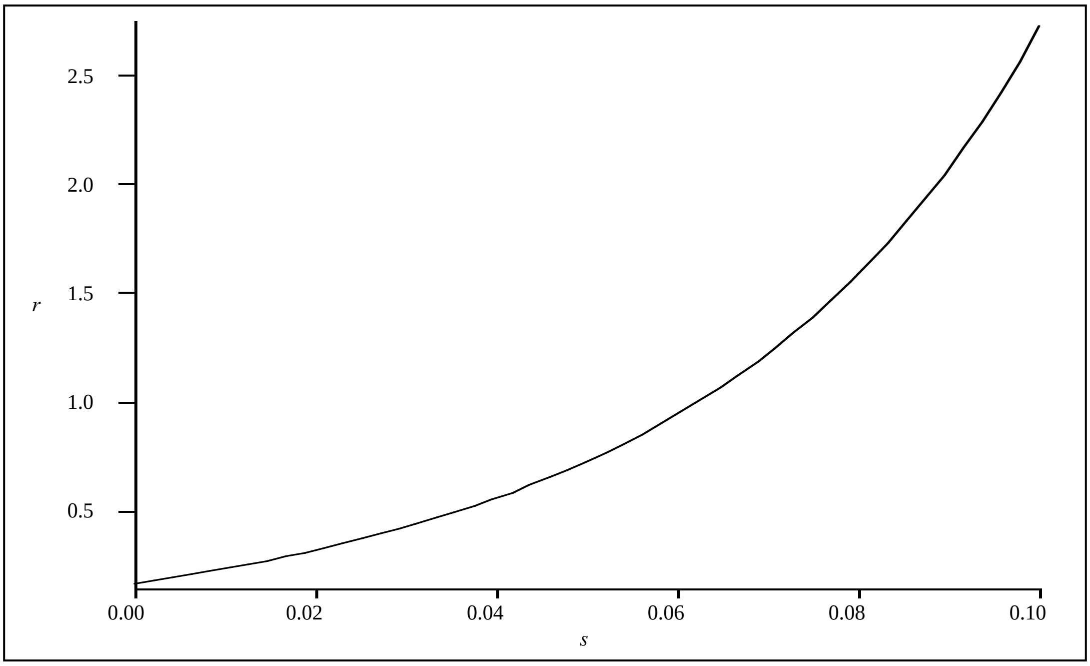
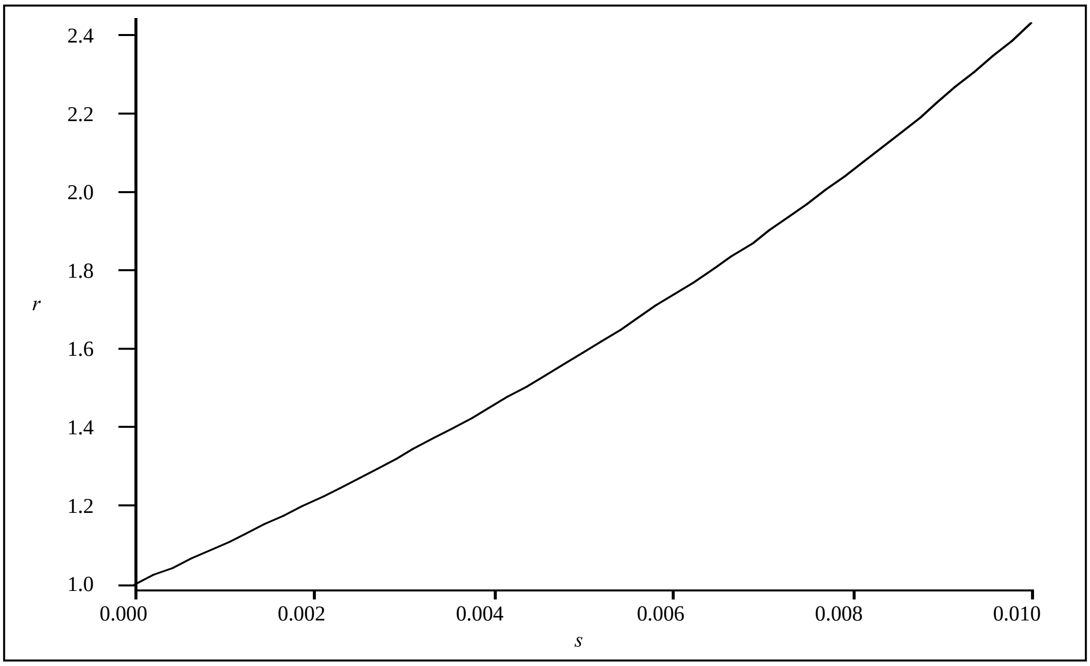


Figure 4-6: *Y*-good Trade Ratio with no Initial Trade in *A*.



C. Numerical Results

Table C-1: With Trade in Both Goods

$N_F = 15, \mu = 2/3$	E_F	p_F	p	V
$\beta = 0.05$				
Initial values	0.2543×10^{-6}	0.0625	0.0625	10.4613
Subsidy used				
$s = 0.1$			0.0563	10.7966
$s = 0.15$			0.0531	9.6040
$\beta = 0.08$				
Initial values	0.1667×10^{-5}	0.1000	0.1000	7.6472
Subsidy used				
$s = 0.1$			0.0900	7.8923
$s = 0.15$			0.0850	7.0206
$\beta = 0.10$				
Initial values	0.4069×10^{-5}	0.1250	0.1250	6.5902
Subsidy used				
$s = 0.1$			0.1125	6.8014
$s = 0.15$			0.1063	6.0502
$N_F = 50, \mu = 2/3$				
$\beta = 0.05$				
Initial values	0.5780×10^{-6}	0.0625	0.0625	12.3825
Subsidy used				
$s = 0.01$			0.0619	12.7255
$s = 0.03$			0.0594	6.9045

The initial value of N is taken to be 5 while the other parameters kept constant are $\alpha = 0.01$, $\sigma = 5$ and $L = 1$.

Table C-2: No initial trade in the *A*-good

$\mu = 0.25$	E_F	p_F	p	V
Initial values	0.5722×10^{-6}	0.0625	0.0625	2.4118
Subsidy used				
$s = 0.01$			0.0619	2.4328
$s = 0.05$			0.0594	1.9754

The initial value of N is taken to be 5, with $N_F = 15$, $\alpha = 0.01$, $\beta = 0.05$, $\sigma = 5$ and $L = 1$.

Chapter 5

Conclusion

5.1. Summary of Main Findings

Market imperfections such as informational coordination failures and scale economies (as we have used in this thesis) provide a basis for governments to intervene in the economy. Evidence such as that documented by Charlton (2003) suggests that there does appear to be relatively widespread use of subsidies as a form of government interventionist policy in the economy. However, the review of the empirical literature by Pack and Saggi (2006) does not suggest that this type of intervention is unambiguously beneficial in terms of welfare: empirical analysis of the use of subsidy provisions on indicators such as factor productivity, growth rates and industry size seem to give no conclusive evidence of its desirability, and suggests instead that a stance of non-intervention may still be still the ideal option.

Thus, our original motivation for the analysis in this thesis was to determine whether there was any theoretical justification for governments to implement subsidy provisions as a form of industrial policy. Our results provide an answer to two key questions we asked at the beginning of this thesis: (i) Why do subsidies continue to be used if empirically, they do not seem to give any beneficial outcomes, (ii) are there any gains to society which are not captured within existing numerical indicators?

Using a non-monetary macroeconomic model with one factor of production, labour, and two produced goods, a homogeneous good and a differentiated commodity, we examined the effects arising from the provision of subsidies by the government to firms in the differentiated goods sector. The differentiated sector is modelled as monopolistically competitive where firms produce a variety of imperfect substitutes with internal increasing returns to scale in production. From our findings, there are three main results which contribute to the existing economic literature.

First, production-cost reducing subsidies to the monopolistically competitive firms lead to either: (i) an increase in the mass of varieties of the differentiated good, (ii) a higher aggregate supply of the differentiated good, or (iii) an increase in both the mass of available varieties and aggregate supply. However, disposable income falls as a result of the use of a proportional income tax to fund the subsidy. In spite of this, we find that the net welfare effect of the subsidy is always positive. This result is robust to the type of subsidy used.

Second, when labour supply is endogenously determined, the tax induces a higher labour supply that offsets the reduction in disposable income. While firms now experience lower production costs due to the subsidy, the increase in the level of economic activity helps in expanding the differentiated goods sector further, which again leads to a higher level of welfare. Furthermore, the elasticity of labour supply does not play a role in determining the optimal policy rule.

Third, an optimal positive subsidy level which raises social welfare to a higher level always exists. This result holds regardless of whether the use of production subsidies is examined within the context of a closed or an open economy. In the closed economy, the elasticity of substitution between varieties plays a key role in determining the level of the optimal subsidy. In the small open economy context, it appears also that the size of the domestic economy relative to the world impacts the magnitude, but not the sign of the optimal subsidy.

In a perfectly competitive environment, economic theory suggests that any form of government intervention in the economy creates distortions and results in an inefficient allocation of resources. Yet, the existence of a deviation away from perfectly competitive settings, such as the presence of a monopolistically competitive goods market, can imply that some form of government intervention may bring about Pareto improvements. A substantial volume of work that explores this comes under the framework of New Keynesian macroeconomics which is characterised by the existence of some form of market imperfection into the analysis. Costa and Dixon (2010) present a recent survey of the developments within this field pertaining specifically to the use of fiscal policy. Their overall conclusions are that some form of relationship does exist between the use of fiscal policy and welfare. However, it is less certain if there are indeed welfare gains arising from the use of government expenditure to influence aggregate demand.

Conversely, the results we have presented in this thesis show that there are always gains to welfare when subsidy provisions are used to influence the level of aggregate supply. Therefore, what we can conclude with some confidence is that, as opposed to the more often analysed Keynesian-type policy of aggregate demand management, subsidies used for aggregate supply management could actually be a plausible and viable alternative policy tool for policy makers to consider instead.

5.2. Policy Applicability

Our results can potentially offer compelling insights which are useful towards policy deliberations. For example, the results from Chapter 2 can help reconcile the apparent discrepancy between the widespread use of subsidies and the empirical results that seem to suggest that they are not beneficial. Subsidies act through a reduction in prices, and lead to increased consumption in quantity or the number of varieties consumed. This subsequently results in a higher level of welfare to the individual, one which is unlikely to be captured in numerical measures such as growth rates or total factor productivity figures. Our results in Chapter 3 offer insights which can be useful towards labour market policy formulation, some of which were previously discussed and we will not repeat them here.

Chapter 4 brings some relevance to the international trade arena. Essentially, while the combination of free trade and abstaining from market intervention do bring Pareto improvements[1], welfare remains lower as compared to what it could be under perfect competition. This sub-optimality is inherent owing to the presence of imperfect markets and provides an incentive for some policy introduction by a benevolent government. That welfare gains are still possible from the use of a production subsidy thus remains a legitimate cause for implementation to correct for this sub-optimality

Overall, the findings from each chapter potentially provides a case on its own for a government to consider enacting some form of industrial subsidy policy.

5.3. Limitations

A note of caution may however be important here. Our results should not be interpreted to mean that subsidies are justified for use or should be used, but that they can potentially yield welfare benefits. In spite of the positive results we have obtained, especially with regards to welfare gains, we cannot claim that our analysis can or ought to be translated into direct policy implementation purposes. We have constructed a very compact and stylised economic model for our analysis. While this is potentially desirable in order to keep our analysis manageable and tractable, this 'reduced form' of analysis also presents several other limitations.

[1] See Krugman (1987) for a discussion on this.

First, labour and firms are assumed to be identical. This can easily make one sceptical about the applicability of our results as observations of wage and firm size differences are regularly observed[2]. Second, our analyses were conducted in a static model, with the condition that the government budget constraint always balances. This can in essence be a very strong and, very possibly, unrealistic assumption in view of large government deficits observed in many countries. Thus, using a model such as that exemplified in Costa and Dixon (2010) and allowing for a one-period budget deficit in our analysis may perhaps offer a more rounded and persuasive argument. Finally, there is no clear quantitative evidence or methodology which can empirically verify our findings at present.

Thus, these aspects need to be taken into account before making any policy recommendations based on the preceding analysis.

5.4. Extensions

The work presented thus far in this thesis offers several potential avenues for further exploration. We list a few here for future consideration.

Two immediate extensions to the theoretical framework developed in this thesis are worth considering for future exploration. The first would entail relaxing the assumption of there being only one factor of production. Taking a lead from McCool (1982), incorporating capital as a factor input into the increasing returns to scale sector with the use of factor-specific subsidies clearly will undoubtedly give added insights. A second extension could consist of including a more realistic form of the labour market. We have followed much of the existing literature by assuming labour

[2] Two possible ways of resolving this could be to incorporate individual firm heterogeneity into the firms' production costs following Montagna (2001) or Melitz (2003), or heterogeneous labour as in Yeaple (2005). See also Molana and Montagna (2000) for an analysis of fiscal policy with firm-heterogeneity in a closed economy framework. However, this could possibly come at a cost of analytical tractability in arriving at explicit solutions as we have obtained.

as homogenous and freely mobile between sectors. Labour supply is either treated exogenously or individuals are free to choose the quantity of labour supplied and with the labour market clearing, the economy is always in a state of full employment. Yet, it is more probable that some degree of matching incompatibility would exist in labour moving between sectors and resulting in some level of involuntary unemployment. Attempting to incorporate this into our model could yield some insightful results. As a start, the recent seminal contribution of Blanchard and Galí (2010) introduced a labour tightness index as an unemployment measure along with hiring costs into a New Keynesian monetary model. Using this, they studied the role of monetary policy as an unemployment determinant or for stabilisation purposes. Considering a similar framework and applying it in our context of subsidy policies could prove a potentially interesting avenue to pursue.

Other areas which could also be particularly interesting include the incorporation of a monetary sector into the model or the migration to other fields such as the New Economic Geography. We did not include a monetary sector, in line with many of the similar New Keynesian macroeconomic models concerned with the use of fiscal policy. For realism however, including a monetary sector will clearly be desirable[3]. Molana (2000) demonstrates that when an individual's utility function includes a preference for money holdings, a non-trivial multiplier effect can still result even though money is neutral. This presents another avenue for future work to advance on. Finally, the open economy formulation in Chapter 4 of this thesis offers a host of other areas to consider. One such would be the field of the New Economic Geography. Matsuyama (1995) covers briefly the implications of factor mobility and firm dispersion across regions as a result of monopolistic competition. This offers

[3] As mentioned by Bénassy (1993), the economy Walras had constructed was a barter economy, but it is quite evident that barter trade hardly ever exists anymore.

promise for a deeper look-in, of which Dupont and Martin (2006) have previously provided a precedent.

5.5. Closing Comments

Chand (2004) argues that micro-founded models such as the one we have constructed in this thesis are not particularly well-suited for policy makers to use in their policy deliberations, owing in part to an aggregation problem of diverse individuals. This raises doubt as to how well suited a 'macroeconomic' model is for policy analysis if it is essentially a 'microeconomic' to begin with. However, he also quotes a statement from Keynes who says that "The object of our analysis is not to provide a machine or method of blind manipulation, which will furnish an infallible answer, but to provide ourselves with an organised and orderly method of thinking out particular problems…"

This is an essential point which we want to emphasise. We feel that the biggest advantage of what we presented in this thesis is the clarity of outcomes. What we have illustrated is that the use of a subsidy can lead the economy to a Pareto-improvement in a constrained optimum. This further gives us confidence that the policy relevance of our findings is very much valid. There will undoubtedly be other barriers such as information asymmetry and various non-economic frictions which will prevent easy translation of our results into policy reality. And it should also be reminded here that the economy will not be in a first-best situation even with the use of the subsidy. But the lack of ambiguity of the results obtained from our analyses can provide a firm base for policy deliberations to start and systematically build upon.

In conclusion, this thesis has examined the effects of providing production subsidies to imperfectly competitive firms within a general equilibrium framework.

Our primary findings show that there are welfare gains to be obtained from the use of production subsidies and we have also derived and showed the existence of a set of optimal policy rules, all of which can hold potential for policy considerations. We have also briefly explored some avenues for future research that offer scope for new and rich insights to be obtained.

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